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HOCHSCHULE WISMAR
UNIVERSITY OF APPLIED SCIENCES
TECHNOLOGY, BUSINESS AND DESIGN

SWIM 2018

11th Summer Workshop on Interval Methods

July 25-27, 2018

Book of Abstracts

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Aim of SWIM

Traditionally, workshops in the series of SWIM provide a platform for both theoretical and applied researchers who work on the development, implementation, and application of interval methods, verified numerics, and other related (set-membership) techniques. Possible areas of usage can be found in the fields of

- the verified solution of initial value problems for ordinary differential equations, differential-algebraic system models, and partial differential equations,

- scientific computing with guaranteed error bounds,

- the design of robust and fault-tolerant control systems,

- the implementation of corresponding software libraries, and

- the usage of the mentioned approaches for a large variety of system models in areas such as control engineering, data analysis, signal and image processing

- ...

Previous Editions of SWIM

The SWIM workshop series was initiated by the French MEA working group on Set Computation and Interval Techniques of the French research group on Automatic Control GDR MACS, where the MEA group especially aimed at promoting interval analysis techniques and applications to a broader community of researchers. Since 2008, SWIM has become an annual keystone event for researchers dealing with various aspects of interval and set-membership methods.

Previous editions of SWIM were held in:

- Manchester, UK in 2017

- Lyon, France in 2016

- Prague, Czech Republic in 2015

- Uppsala, Sweden in 2014

- Brest, France in 2013

- Oldenburg, Germany in 2012

- Bourges, France in 2011

- Nantes, France in 2010

- Lausanne, France in 2009

- Montpellier, France in 2008

In view of the fact that the city of Rostock will celebrate its 800th anniversary in 2018 and that the University of Rostock (founded in 1419) can look back to an almost 600-year time span of tradition and innovation, we cordially invite you to participate in our workshop with the following interesting contributions.

SWIM 2018

11th Summer Workshop on Interval Methods

July 25-27, 2018

Tuesday, July 24

18:00	Welcome Reception

Wednesday, July 25

8:30 – 9:00	Opening	14:15 – 16:15	Algorithmic Differentiation and Mathematics
9:00 – 10:30	Hybrid Systems		
10:30 – 10:50	Coffee Break	17:10	Meeting for Evening Tour
10:50 – 12:20	Control Design and Verification	17:30 – open end	Evening Tour and Dinner
12:20 – 14:15	Lunch		

Thursday, July 26

8:30 – 10:00	PDEs: Verification and Modeling	14:15 – 15:15	Uncertainty Modeling 2
10:00 – 10:20	Coffee Break	15:15 – 16:15	Geodetic Systems
10:20 – 12:20	Uncertainty Modeling 1	16:15 – 16:45	Coffee Break
12:20 – 14:15	Lunch	16:45 – 18:15	Robotics

Friday, July 27

8:30 – 10:30	Estimation
10:30 – 10:50	Coffee Break
10:50 – 12:50	Uncertainty Modeling 3
12:50 – 13:00	Closing and Announcement for SWIM 2019
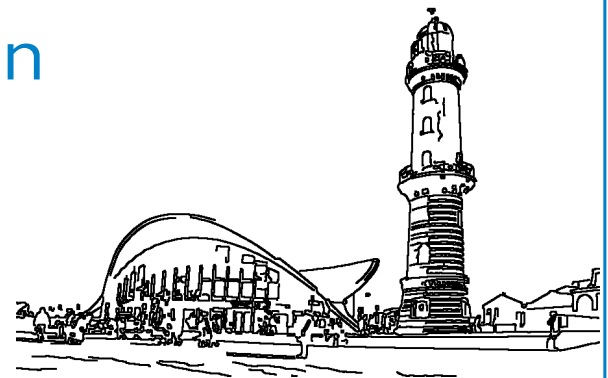
Tuesday, July 24

18:00	Welcome Reception

Wednesday, July 25

| 8:30 – 9:00	Opening	
Hybrid Systems		
9:00 – 9:30	Optimal Switching Instants for the Control of Hybrid Systems	Olivier Mullier
9:30 – 10:00	On Mode Discernibility and Bounded-Error State Estimation With Hybrid Systems	Nacim Ramdani
10:00 – 10:30	Interval Estimation for Continuous-Time LPV Switched Systems	Chaima Zammali
10:30 – 10:50	Coffee Break	
Control Design and Verification		
10:50 – 11:20	Static Output Feedback Control by Interval Eigenvalue Placement Using Quantifier Elimination	Klaus Röbenack
11:20 – 11:50	A Set Membership Approach to Parametric Synthesis of Reliable Stabilising Controllers	Nacim Ramdani
11:50 – 12:20	Guaranteed SLAM – Practical Considerations	Eduard Codres
12:20 – 14:15	Lunch	
Algorithmic Differentiation and Mathematics		
14:15 – 14:45	AdolC4Matlab – An Interface between MATLAB and ADOLC for Applications in Nonlinear Control	Mirko Franke
14:45 – 15:15	Data, Langrangian, Action! Simulating Mechanisms Direct From a Text File	John D. Pryce
15:15 – 15:45	Accurate and Reproducible Matrix Multiplication	Katsuhisa Ozaki
15:45 – 16:15	An Interval Median Algebra	Stavros P. Adam
17:10	Meeting for Evening Tour	

Thursday, July 26

	PDEs: Verification and Modeling	
8:30 – 9:00	Verified Solution of an Optimal Control Problem for Elastic Rod Motions Based on the Ritz Method	Georgy Kostin
9:00 – 9:30	Computer-Assisted Existence Proofs for One-Dimensional Schrödinger-Poisson Systems	Jonathan Wunderlich
9:30 – 10:00	Traveling Waves in a Chemotaxis Model	Georgia Kokkala
10:00 – 10:20	Coffee Break	
	Uncertainty Modeling 1	
10:20 – 10:50	Reliable Visual Analytics as Part of a Process Quality Assessment	Wolfram Luther
10:50 – 11:20	Interval Computations in C++ 20	Walter F. Mascarenhas
11:10 – 11:50	Safe Approximation of Probabilities	Walid Taha
11:50 – 12:20	Statistics for Imprecise Data	Marco De Angelis
12:20 – 14:15	Lunch	
	Uncertainty Modeling 2	
14:15 – 14:45	Measurement Errors in Magnetic Resonance Imaging of Fluid Flows	Martin Bruschewski
14:45 – 15:15	Reachability Analysis for a Class of Uncertain Discrete-Time Systems*	Nacim Meslem *presented by N. Ramdani
	Geodetic Systems	
15:15 – 15:45	Interval-Based Global Optimization for Geodetic Network Adjustment Procedures	Tomke Lambertus
15:45 – 16:15	Guaranteed Bounding Zones for GNSS Positioning by Geometrical Constraints	Hani Dbouk
16:15 – 16:45	Coffee Break	
	Robotics	
16:45 – 17:15	RGB-Laser Odometry Under Interval Uncertainty for Guaranteed Localization	Raphael Voges
17:15 – 17:45	Bringing Interval Methods in ROS, the Robot Operating System	Vincent Drevelle
17:45 – 18:15	Cooperative Localization of Drones by Using Interval Methods	Ide Flore Kenmogne

Friday, July 27

Estimation		
8:30 – 9:00	A Robust Fault Detection Method Using a Zonotopic Kaucher Set-Membership Approach – Application to a Real Single-Tank Process	Stefan Schwab
9:00 – 9:30	Interval State Estimation With Data Association	Luc Jaulin
9:30 – 10:00	Observability of Nonlinear Systems Using Interval Analysis	Thomas Paradowski
10:00 – 10:30	State Estimation and Control Design for Cooperative Dynamic Systems	Andreas Rauh
10:30 – 10:50	Coffee Break	
Uncertainty Modeling 3		
10:50 – 11:20	Transformation of Dynamic Systems Into a Cooperative Form to Exploit Advantages in Interval-Based Controller Design	Julia Kersten
11:20 – 11:50	Reliable Propagation of Time Uncertainties in Dynamical Systems	Simon Rohou
11:50 – 12:20	VERICOMP: Comparing and Recommending Verfied IVP Solvers	Ekaterina Auer
12:20 – 12:50	Inner and Outer Approximation of the Viability Kernel for a Bounded Uncertain System	Mario Martinez Guerrero
12:50 – 13:00	Closing and Announcement for SWIM 2019	
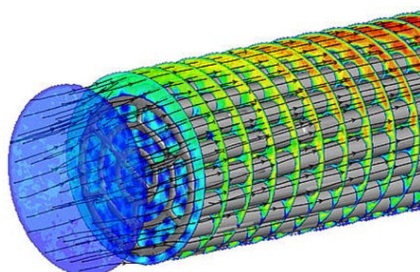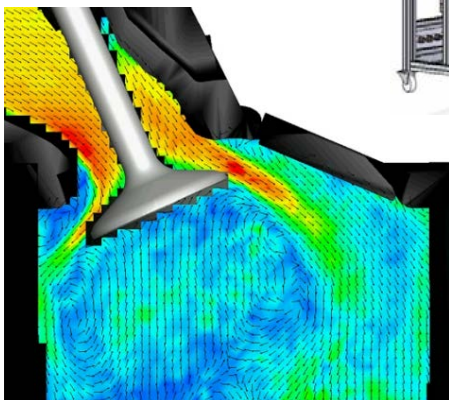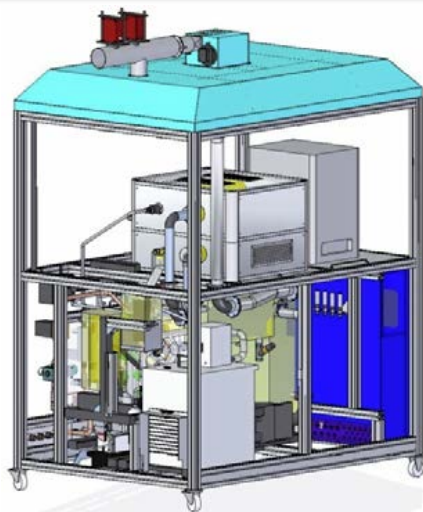
Research activities at the Chair of Mechatronics and at the Institute of Fluid Mechanics at the University of Rostock
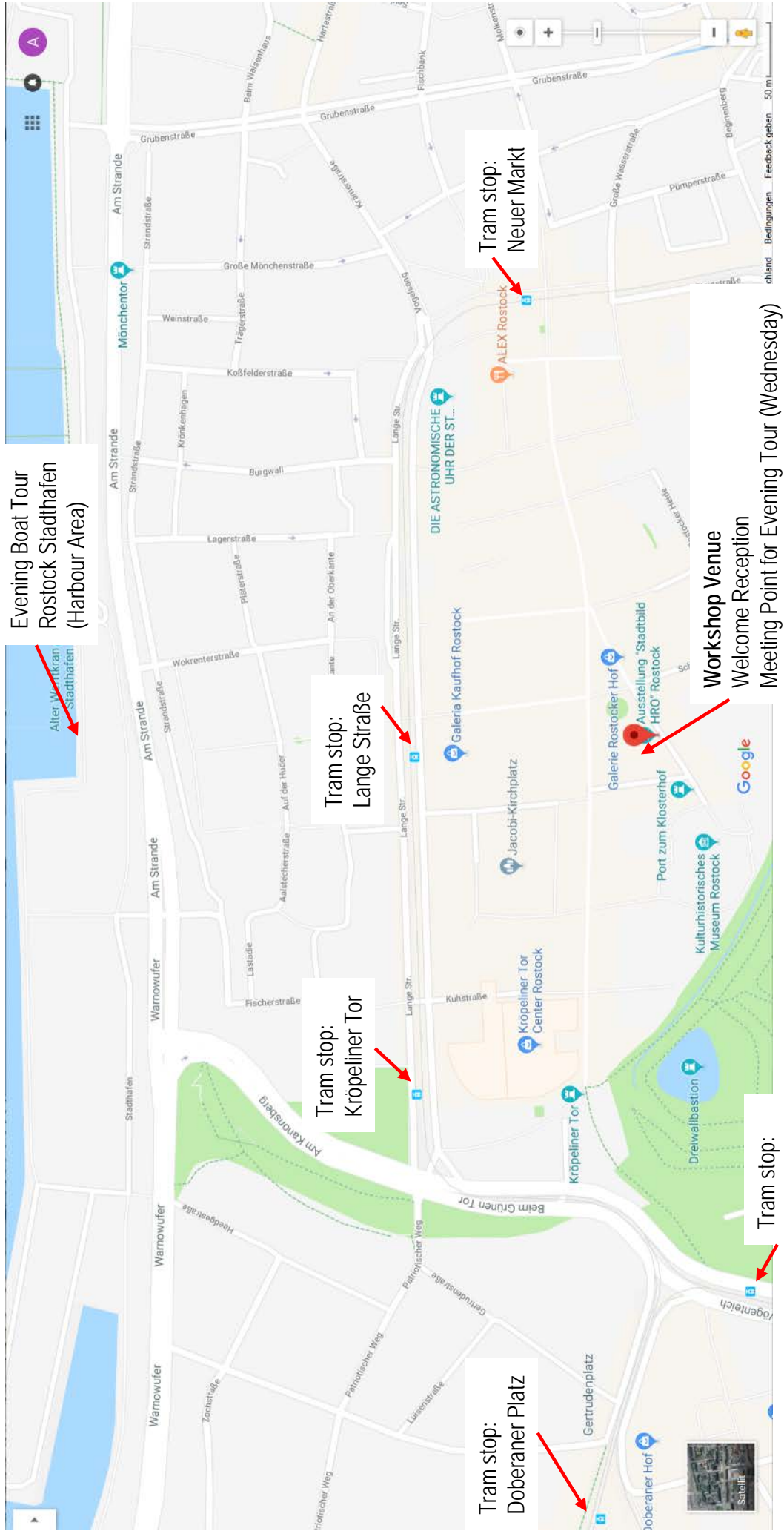
Venue and Public Transport Information

Web page of the Rostock public transport system
www.rsag-online.de

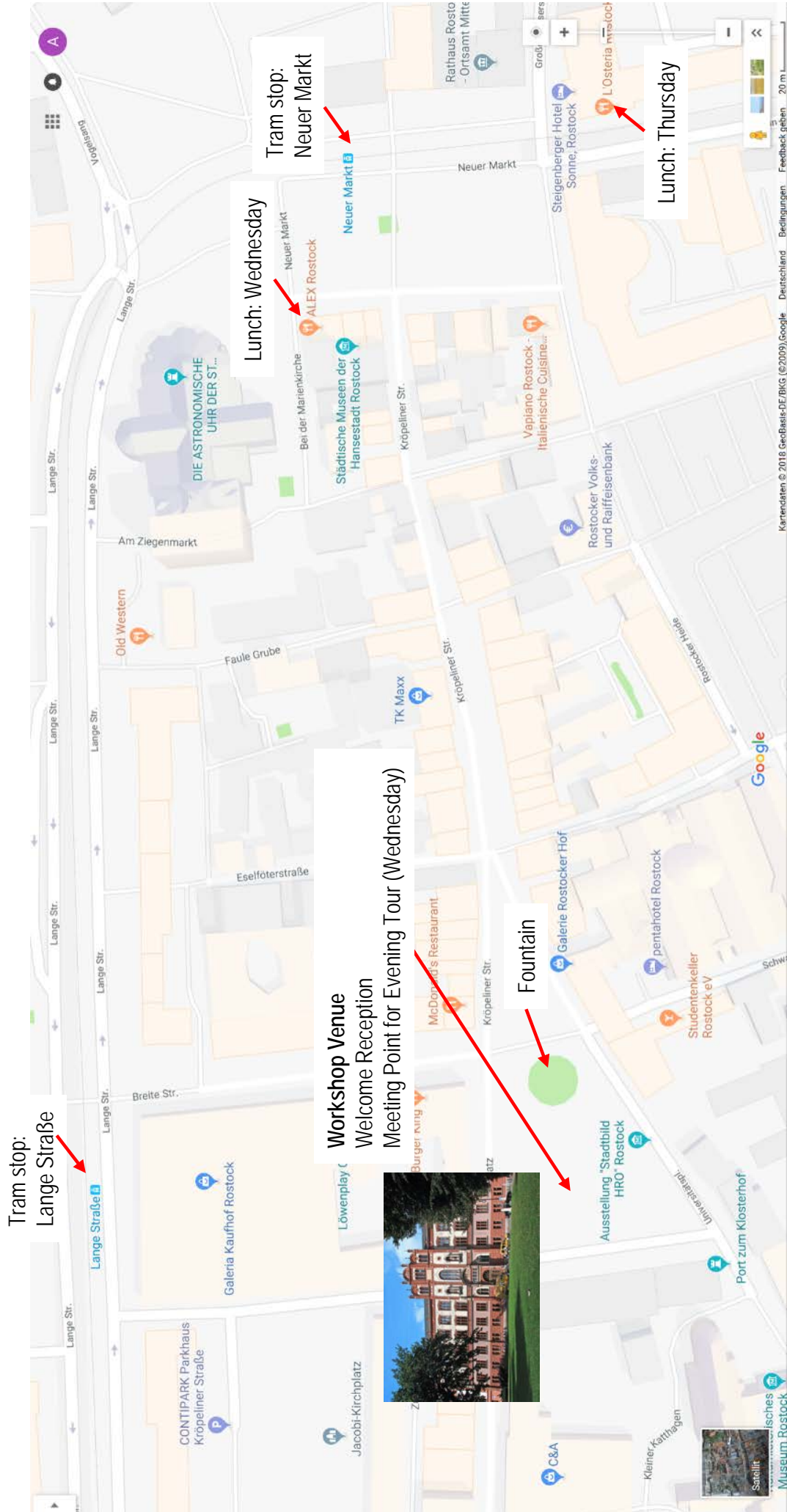Electronic tickets available via VVW-App (Android&iOS)
Alternatively, ticket machines are available in the trams and at the tram stops
For distances up to 4 stops (Kurzstrecke): 1,50€, otherwise: 2,10€ (Einzelfahrkarte)

Evening Boat Tour
Rostock Stadthafen
(Harbour Area)

Tram stop:
Neuer Markt

Tram stop:
Lange Straße

Workshop Venue
Welcome Reception
Meeting Point for Evening Tour (Wednesday)

Tram stop:
Kröpeliner Tor

Tram stop:
Schröderplatz

Tram stop:
Doberaner Platz

Venue and Public Transport Information

Contents

Optimal Switching Instants for the Control of Hybrid Systems

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Keywords: Hybrid Systems, Reachability, Optimization

Problem Statement

In this talk, we will consider the problem of determining optimal switching instants for the control of hybrid systems under reachability constraints. First, we define an n-mode dynamical system

$$(\mathcal{S}_i)\left\{\dot{x} = f_i(x) \quad , \quad x(t_i) = x_i\right\}$$

in the time interval $[t_i, t_{i+1}]$ with $f_i : \mathbb{R}^m \to \mathbb{R}^m$ where $x_i \in \mathbb{R}^m$ is the initial condition for all modes $0 \leqslant i \leqslant n-1$. Apart from x_0 that is fixed, x_i is taken as the solution at time t_i of the previous dynamical system $(\mathcal{S}_{i-1})$. This sequence of dynamical systems corresponds to the switching of control law. Our problem can be modeled using the following optimization problem

$$\begin{bmatrix} \max_{t_1,\ldots,t_{n-1}} & g(x(\tau)) & \text{(cost function)} \\ \text{s. t.} & \forall 0 \leqslant i \leqslant n-1, (\mathcal{S}_i) & \text{(dynamical constraint)} \\ & h(x(\tau)) > 0 & \text{(reachability constraint)} \\ & \tau \in [t_{n-1}, t_n] & \end{bmatrix} \quad (1)$$

with the decision variables $t_1, \ldots, t_{n-1} \in \mathbb{R}_+^n$ the search space for the different times; $g : \mathbb{R}^m \to \mathbb{R}$ the cost function on the state variable at given time $\tau \in [t_{n-1}, t_n]$, some constraints defined by the dynamical systems $(\mathcal{S}_i)$ and the times t_i; a reachability constraint using $h : \mathbb{R}^m \to \mathbb{R}$.

Approach

The optimization problem (1) is cast into a global optimization problem with differential constraints, where validated simulation techniques [1] and dynamic time meshing are used for its solution.

Example of the Goddard's Rocket

The Goddard problem [2] models the ascent of a rocket through the atmosphere. The rocket has to reach a given altitude while consuming the smallest amount of fuel. Physically, the optimal solution is a bang-singular arc-bang controller following the steps: 1) full power to break out 2) an increasing function to compensate for the drag effect and 3) turn off the engine and continue with the impulse. The question is when to switch from one dynamics to the following one.

Our approach provides the controller given in figures below, results agreed with [3] such as $t_1 = 0.019$, $t_2 = 0.063$ for a mass $m = 0.6273$.
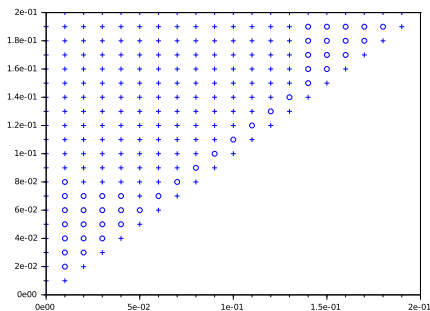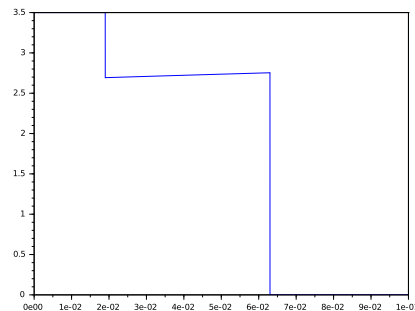
Mesh for t_2 w.r.t. t_1 Optimal controller

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On Mode Discernibility and Bounded-Error State Estimation with Hybrid Systems

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Keywords: Hybrid Systems, Estimation, Reachability, Identifiability

Introduction

State estimation is a key engineering problem when addressing control or diagnosis issues for complex dynamical systems. The issue is still challenging when the latter systems need to be modelled as hybrid discrete-continuous dynamics, which is true for many complex and safety-critical systems. In this talk, we will review our latest results regarding nonlinear hybrid state estimation in a bounded-error framework using reliable and robust methods [1].

Main Results

Hybrid state estimation aims at reconstructing both the discrete mode, hence the switching sequence, and the associated continuous state variables, based on a set of possibly discrete-time measurements, the knowledge of the hybrid model, and assumptions about the uncertainties and perturbations acting on the system.

We first establish a testable condition for current mode location discernibility. If the hybrid system's active operation mode was known, the estimation of the continuous component of the hybrid system would

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merely make use of the existing set membership estimation (SME) algorithms for continuous systems. Therefore, the main ingredient of our SME for hybrid systems is the ability to distinguish the currently active location mode from the observation of the input-output behaviour. To the best of our knowledge, the observability and detectability of hybrid systems have been studied only for linear switching systems (see [1] and the references therein). In [1], we introduced a new computable condition for analyzing mode discernibility for the general class of nonlinear hybrid systems. We say that two location modes are discernible if there exists a control making it possible to distinguish them by their outputs. In the case of autonomous systems, the output trajectories must differ at some point in time. Then, using a one-parameter-tuned composite continuous model, we show that the identifiability of the tuning parameter implies current mode discernibility.

In a second stage, we build our hybrid state estimator which relies on a prediction-correction approach. For the prediction step, we use the hybrid reachability method developed in [2], which combines interval Taylor methods and zonotope enclosures to bound the solution set of an IVP ODE. The correction step relies on the algorithm of [3] to compute the intersection between a zonotope and a strip.

An example inspired from vehicle suspension systems is presented.

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Interval Estimation for Continuous-Time LPV Switched Systems

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Keywords: Interval Observer, Continuous-Time LPV Switched Systems, Cooperative Dynamics, Polytopic Parameter Dependence

Abstract

The theme of diagnosis of industrial systems is a major technological challenge. In order to take into account this reality, the synthesis of fault prevention, detection and localization techniques must be highlighted. Mainly in the automotive, metallurgy and aerospace industries, among the most encountered systems, we found the hybrid one. In this paper, we are interested in the class of switching systems which is the most important class of hybrid systems. These systems involve both continuous and discrete dynamics. They consist of a finite number of continuous dynamical subsystems combined with a discrete rule that operates switching between these subsystems [1]. In the last two decades, this class of systems has been widely studied in the frame of stability, stabilization, observation and diagnosis problems. In order to design a stabilizing control law or a diagnosis procedure, many existing works consider that the evolution of the system state can be known or measured. However, the increasing complexities of industrial systems lead to study more complex nonlinear systems where the state cannot be directly measured [2]. This problem is widespread. Usually, for nonlinear systems, state estimation methods are based on an approximate linearization which can lead to an unprecedented level of obstruction in practice. Accordingly, a broad class of nonlinear systems can be represented in an LPV form [3]. In the literature,

several approaches of state estimation based on interval methods for LPV systems provide good results even when the system is affected by disturbances and/or uncertainties. For an LPV switched system, interval observers can be designed to estimate lower and upper bounds of continuous states at each time instant [4], when the uncertainties and disturbances are assumed to be unknown but bounded with known bounds [5]. Our contribution deals with the interval state estimation for LPV switched systems with measured polytopic parameter dependence when the switching signal is assumed to be known. Considering that the measurement noise and the state disturbance are unknown but bounded, and that the dynamics of the system is described by a convex combination, lower and upper bounds of the state are therefore determined. An interval observer is designed to guarantee both stability and cooperativity of the observation errors. The efficiency of the proposed interval observer is highlighted through simulation results.

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Static Output Feedback Control by Interval Eigenvalue Placement Using Quantifier Elimination

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Keywords: Stabilization, LTI Systems, Eigenvalue Placement

Introduction

The design problem of static state feedback of LTI systems has been solved decades ago. In combination with a state observer one obtains a dynamic output feedback control law. Contrary to that, the design problem of static output feedback is significantly more challenging, especially if prescribed eigenvalue requirements are considered [1, 2].

Eigenvalue Placement

We consider the state-space system

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad u = Ky \tag{1}$$

with the state x, the input u, the output y and the matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{r \times n}$. The system is controlled by a static output feedback controller with the gain matrix $K \in \mathbb{R}^{m \times r}$. The closed-loop characteristic polynomial has the form

$$\mathrm{CP}(s) = \det\left(sI - (A + BKC)\right) = a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + s^n,$$

where the coefficients depend on the entries k_{ij} of the matrix K. The gain matrix K should be computed such that the closed-loop system has a prescribed characteristic polynomial

$$\text{CP}^*(s) = (s - s_1) \cdots (s - s_n) = a_0^* + a_1^* s + \cdots + a_{n-1}^* s^{n-1} + s^n.$$

The existence of an appropriate gain matrix K can be stated as

$$\exists k_{11} \cdots \exists k_{mr} : \ a_0 = a_0^* \wedge \ldots \wedge a_{n-1} = a_{n-1}^*. \tag{2}$$

In a similar manner, the stabilizability of system (1) can be verified using inequality constraints on the coefficients $a_0, \ldots, a_{n-1}$ resulting from classical stability criteria such as Routh, Hurwitz, or Liénard-Chipart [3, 4, 5]. For example, if the state-space of system (1) has dimension $n = 3$, the stabilizability by static output feedback is stated as

$$\exists k_{11} \cdots \exists k_{mr} : \ a_0 > 0 \wedge a_2 > 0 \wedge a_1 a_2 - a_0 > 0. \tag{3}$$

A stable system can still have a complex conjugate pair of eigenvalues. In the time domain, such an eigenvalue configuration corresponds to declining oscillations. To avoid these oscillation, one could demand a purely real stable eigenvalue configuration. The stabilizability by real eigenvalues can be formulated as

$$\exists s_1 \cdots \exists s_n \exists k_{11} \cdots \exists k_{mr} : \quad \begin{aligned} & s_1 < 0 \wedge \cdots \wedge s_n < 0 \wedge \\ & a_0 = a_0^* \wedge \ldots \wedge a_{n-1} = a_{n-1}^* \end{aligned} \tag{4}$$

corresponding to a placement into the interval $s_1, \ldots, s_n \in (-\infty, 0)$.

Quantifier Elimination

The expressions (2) to (4) contain quantifiers ($\exists$). These formulas can be transformed into quantifier-free equivalents using *quantifier elimination* methods [6]. To illustrate the concept we consider the question, under which conditions on the parameters a quadratic equation has at least one real root. This problem can be formulated as

$$\exists x : \ x^2 + px + q = 0$$

with the quantified variable x and the free variables p, q (parameters). Quantifier elimination yields the equivalent quantifier-free expression

$$p^2 - 4q \geq 0.$$

There are several open source as well as commercial software packages for quantifier elimination available [7, 8].

This contribution deals with the static output feedback problem using quantifier elimination. The stabilization problem has been discussed in [3, 9]. We consider the eigenvalue placement into open, half-open and closed intervals [10]. Our approach is illustrated on some example systems.

Acknowledgement

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A Set Membership Approach to Parametric Synthesis of Reliable Stabilising Controllers

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Keywords: Nonlinear Systems, Reachability, Set Integration, Set Inversion

Introduction

The dynamical behaviour of physical or industrial systems is usually described by complex mathematical models, which include several types of non-linearity and stiffness. However, almost all non-linear control design methods are based on simplified models, which satisfy some mathematical assumptions. Therefore, in practice, there exists a gap between the thorough modelling of the actual system and the simplified modelling used to design control laws. As a consequence, the expected performance of the control law can no longer be guaranteed if used with the actual system without further investigation. In this talk we will discuss the potential of our recent contribution in the field [1].

Main Results

This work focuses on the design of reliably stabilizing controllers for complex non-linear systems.

For a given bounded domain of initial state vectors and a given parametric feedback control structure derived from a simplified model, the control parameters are tuned such that the complex system reaches

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a specified target set in a finite-time. Moreover, under the identified control law the non-linear system must be locally asymptotically stable over the target set. Note that the finite-time reachability specification is equivalent to the desired settling time performance for feedback systems and the width of the target set can be considered as the required steady-state error performance specification.

A set membership (SM) algorithm is developed in order to characterize, in a rigorous way, the feasible set of the control parameters. Interestingly, this SM algorithm can also be used to validate or certify a given stabilizing controller designed from a nominal model of a complex non-linear system.

First, the control design problem is reformulated as a parameter identification issue in an unknown-but-bounded error framework. Then, a new dual set integration method for reachability computation is introduced; it combines an interval Taylor method for solving an IVP for a set of ODEs and a bounding method for order-preserving monotone systems. Finally, the latter dual method is used in conjunction with set inversion techniques via interval analysis, in order to develop a SM parameter estimation algorithm to solve the control design problem.

The identified feedback control must achieve two aims. The first aim is that, starting from a given bounded set of initial states, the state trajectories generated by the controlled non-linear system have to reach in a finite-time a desired target set. The second aim is to ensure asymptotic stability of the controlled system over the target set. The effectiveness of the proposed method is illustrated through a complex non-linear system.

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Guaranteed SLAM – practical considerations

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Abstract

Mobile robots have constantly been developing and gaining capabilities during the past few decades, yet some of the most important requirements, such as localisation and mapping, are still under research. Building a reliable map of the environment is a critical task. Firstly because the mobile robot needs to have safe navigation in unknown environments; and secondly because in many cases an accurate map is needed for future use when the environment is unknown.

Many authors propose interval SLAM (Simultaneous Localisation and Mapping) for solving the drawbacks of the probabilistic methods (they need specific noise distributions, model linearisation, they are not guaranteed etc.). Most of the interval approaches try to deal with model nonlinearities or sensor noise in a guaranteed way: SLAM for an underwater robot with sonar [1], range only SLAM [2], SLAM using Kinect sensor with point features [3]. The convergence of the map for a guaranteed SLAM approach is proven in [4].

One of the biggest drawbacks when using interval methods is that, in many cases, they can be pessimistic. A solution for dealing with this drawback is to use more data and consequently to add more constraints when solving the problem at hand as a CSP (Constraint Satisfaction Problem). Another issue with the existing guaranteed SLAM

approaches is that they rely on existing methods or algorithms to detect features in the data provided by the sensors. This approach decreases the number of constraints to be used for solving the SLAM CSP which is benneficial for the computation time, but at the same time it creates a different set of problems because the uncertainty of the detected features cannot be estimated in a straightforward manner.

To adress the drawbacks presented above, a SLAM method which takes into account all sensor measurements and generates a CSP is proposed. A non-holonomic robot with a 2D LiDAR sensor is used to test the proposed method. In this approach two main difficult problems have to be adressed:

- The data asocciation problem has to be solved for each sensor measurement in order to be able to contact the CSP;

- Computation time has to be small enough such that the CSP is solved in real time.

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AdolC4Matlab – An Interface Between MATLAB and ADOL-C for Applications in Nonlinear Control

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Introduction

Many algorithms in nonlinear control require the computation of derivatives like Jacobians, Hessians or certain Lie derivatives [1]. Especially for complex systems, the symbolic computation can result in a time and memory consuming task as the derivative order increases. This effect may be avoided when algorithmic differentiation is used instead of the symbolic computation of derivatives.

Algorithmic Differentiation

The method of algorithmic differentiation is applicable to functions given as algorithms. As for symbolic differentiation such a function will be separated by the compiler into its elementary subexpressions for which the corresponding differentiation rules are applied. The main difference to the symbolic computation is that these subexpressions as well as their derivatives are evaluated numerically and passed on as floating point numbers instead of symbolic expressions. Thus, an exponential increase in computing time and memory consumption is prevented.

The open source algorithmic differentiation toolbox ADOL-C [2] is designed for the computation of first and higher order derivatives of

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vector functions that predestines it for tasks in control engineering. It is written in C/C++ and can be easily connected to MATLAB using so called MEX functions.

Functionality of the Interface

Functions that are going to be used for differentiation operations first are passed to an automated procedure to build a MEX function, which, when executed, generates so called tapes. This is a special data set containing all necessary information of a considered function which then ADOL-C uses for derivative computation later.

Furthermore, the interface provides a series of ready to use MEX functions accessing the tapes for their operations, e.g., to get the gradient, Jacobian or Hessian of a function or various Lie derivatives along vector fields. Part of this functions are MATLAB MEX wrappers around the corresponding ADOL-C functions. Beside this basic functionality there is also a more sophisticated and control engineering related part. This contains the direct computation of gains for controllers as the exact input-output linearization or observers such as the extended Luenberger observer.

The structure of the interface also allows an easy extension by user defined functions. In fact, the interface itself is not constrained to MATLAB but may also be used by the open source tool Octave.

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Data, Lagrangian, Action!

Simulating mechanisms direct from a text file

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Introduction

This is about linkage mechanisms, built from rigid bodies, springs etc., connected by joints of various kinds. For simulation, a Lagrangian approach to such systems is popular because of its economy and flexibility.

The economy is because by d'Alembert's principle a Lagrangian function L, in contrast to direct use of Newton's laws, can omit mention of forces that do no work. E.g., for L not to mention the string tensions either side of a pulley amounts to declaring the pulley is frictionless.

The flexibility is because the system can be described in any vector $\mathbf{q}$ of "generalised coordinates" that specify its position.

If $\mathbf{q}$ is such that constraint equations are involved (say, a 2D simple pendulum in terms of x, y coordinates) the resulting *Euler–Lagrange equations* of motion form an index-3 differential-algebraic equation (DAE) system. If not (say, the pendulum in terms of its angle with the vertical), they reduce to an ODE. Since DAEs are thought difficult, much work has gone into ways of describing a system by coordinates, usually angles, that give an ODE.

However our `C++` DAE solver DAETS has no problem with the index-3 form. Our first contribution is a "Lagrangian facility" that converts code for L, whether constrained or not, direct to the equations

of motion, in a form DAETS then solves. DAETS's builtin automatic differentiation (AD) package `FADBAD++` does the relevant $\partial/\partial q_i$, $\partial/\partial \dot{q}_i$ and $\mathrm{d}/\mathrm{d}t$ at run time—no computer algebra manipulation involved.

Next, we have shown a systematic way to model dynamics of a rigid body (in any number n of dimensions) in terms of cartesian coordinates of n "reference points" $\mathbf{r}_j$ on it. The key computational tool is QR-factorisation. Hence a mechanism can be given a $\mathbf{q}$ made of suitable $\mathbf{r}_j$ on the moving parts (plus, a torque needs an associated turn angle, to model it as a conservative force). The constraints define how it is jointed. This usually gives a simple, readable cartesian Lagrangian.

Our second contribution is to build a "mechanism facility" that reads a formalised text description of a mechanism and converts this (again at run time) to a cartesian L that the Lagrangian facility then solves. The description is written in YAML, a Python-style data serialisation language. At present the facility is restricted to a class of mechanisms in 2D: parts can be rigid bodies, particles or springs, acted on by constant forces or torques, or gravity; joints are pin-joints or linear sliding constraints. Widening this class, and extending to 3D, is a matter of detail and not of concept.

I aim to show how for small mechanisms one can go from a problem description in a text editor, to a Matlab animation, in a few seconds.

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Accurate and Reproducible Matrix Multiplication

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Introduction

Reproducibility is an important topic in the field of high-performance computing. Reproducibility means that a bitwise identical floating-point result is always obtained for the same inputs in different computational environments. We propose reproducible algorithms that produce accurate numerical results for matrix multiplication based on a previous study [1,2].

Error-Free Transformation and Main Results

Let $\mathbb{F}$ be a set of floating-point numbers, as defined by IEEE 754 [3]. The notation $\texttt{fl}(\cdot)$ indicates a computed result, i.e., all operations in the parenthesis are evaluated via floating-point arithmetic. For $A \in \mathbb{F}^{m \times n}$ and $B \in \mathbb{F}^{n \times p}$, we aim to obtain an accurate numerical result for the matrix product AB. As given in a previous study [1], we divide A and B into an unevaluated sum of k floating-point matrices, such that

$$
\begin{aligned}
A &= A^{(1)} + \cdots + A^{(k-1)} + \underline{A}^{(k)}, \quad A^{(i)}, \underline{A}^{(k)} \in \mathbb{F}^{m \times n}, \\
B &= B^{(1)} + \cdots + B^{(k-1)} + \underline{B}^{(k)}, \quad B^{(i)}, \underline{B}^{(k)} \in \mathbb{F}^{n \times p},
\end{aligned}
$$

where $1 \leq i \leq k - 1$. Then, we compute AB using the following form:

$$AB = \sum_{i+j \leq k} A^{(i)} B^{(j)} + \sum_{i=1}^{k-1} A^{(i)} B^{(k-i+1)} + \underline{A}^{(k)} B. \qquad (1)$$

In (1), $\mathtt{fl}(A^{(i)} B^{(j)}) = A^{(i)} B^{(j)}$ is satisfied for $i + j \leq k$. The computed result $\mathtt{fl}(A^{(i)} B^{(j)})$ does not depend on the computational environment. Therefore, if we compute AB as

$$AB \approx \sum_{i+j \leq k} \mathtt{fl}(A^{(i)} B^{(j)}) \qquad (2)$$

and if the sum in (2) is computed in a particular order, we obtain a bitwise identical result in any computational environments. However, if we evaluate (1) via floating-point arithmetic, the computational results will depend on the order of evaluation, because in many cases

$$\mathtt{fl}(A^{(i)} B^{(k-i+1)}) \neq A^{(i)} B^{(k-i+1)}, \quad \mathtt{fl}(\underline{A}^{(k)} B) \neq \underline{A}^{(k)} B, \ 1 \leq i \leq k - 1.$$

We propose a method that provides a reproducible result on MATLAB, Scilab and Octave based on (1) through the suitable application of blockwise matrix multiplication under several assumptions. Finally, we introduce reproducible algorithms for interval matrix multiplication.

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An Interval Median Algebra

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Keywords: Intervals, Median Algebra, Median Structure

Introduction

Intervals and interval computation owe much to the seminal work of Sunaga [1] who defined intervals and interval arithmetic as a result of an Interval Algebra. As discussed in [2], the set of real intervals $\mathbb{IR}^*$ with the operations Min, Max is a distributive lattice with a number of interesting algebraic properties. In this paper, we are interested in endowing this lattice of intervals with a median structure thus obtaining a median algebra. We show how this algebra is defined, its properties, its compatibility with the interval arithmetic and how intervals (proper, improper and modal) are treated in this framework. Finally, we discuss some theoretical issues as well as some potential applications.

Background and Hypotheses

Since the early times of interval computation [1], the concept of interval and related arithmetic gave rise to the elaboration of a significant number of theoretical and computational concepts and tools under the

title of interval analysis as well as important developments in various scientific and engineering fields. Hence, the initial interval concepts were gradually enriched with the concepts of improper intervals and the related Kaucher arithmetic as well as modal intervals together with related operations. In addition to the above, one should mention set-membership approaches which adopted an interval based view of sets by means of unions of axis aligned boxes. This view, which can be significantly represented by SIVIA [3] and its ramifications, resulted in important research and development work in the area of control systems and their applications. Among all the important foundational approaches it seems that the work of Sunaga [1] was the first that introduced an algebraic structure for the intervals and yet proved its contribution to the definition of interval arithmetic operations and to the perspective of using intervals in numerical analysis.

In algebra the concept of median structure and the resulting median algebra constitutes a well established algebraic construct with constantly increasing appeal from both theoretic and practical points of view of algebraic notions [4,5]. A median algebra is a ternary algebraic structure consisting of a set M together with a ternary operation $(\alpha, \beta, \gamma) \mapsto (\alpha\beta\gamma)$ on M such that the following axioms hold:

- $(\alpha\alpha\beta) = \alpha$ (Idempotency)

- $(\alpha\beta\gamma) = (\beta\alpha\gamma) = (\alpha\gamma\beta)$ (Symmetry)

- $(\alpha\beta(\gamma\delta\epsilon)) = ((\alpha\beta\gamma)\delta(\alpha\beta\epsilon))$ (Distributivity)

If M is a distributive lattice $(M, \vee, \wedge)$ one can define a ternary operation $m : M^3 \to M$ such that for $\alpha, \beta, \gamma \in M$

$$m(\alpha, \beta, \gamma) = (\alpha \vee \beta) \wedge (\alpha \vee \gamma) \wedge (\beta \vee \gamma),$$

where the symbols $\wedge$ and $\vee$ denote the *join* and *meet* operations of the lattice M. This operation is referred as the median ternary operation in M and satisfies the axioms above. Moreover, this ternary operation is self dual in the sense that

$$(\alpha \vee \beta) \wedge (\alpha \vee \gamma) \wedge (\beta \vee \gamma) = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vee (\beta \wedge \gamma).$$

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Since every chain is a distributive lattice, so are the real numbers and therefore $\mathbb{R}$ is a median algebra. Median structures have been successfully applied in various areas of interest such as metric spaces giving rise to median spaces and measured wall spaces, consensus theory, taxonomy, majority decision making, median graphs and trees which often occur as almost-natural representations of structured data in various applications of machine learning, etc.

Main Results

In this paper, we show how the set of real intervals $\mathbb{IR}$ becomes a median algebra and so does its extension $\mathbb{IR}^*$ of modal intervals. Using some of the foundational concepts defined in [1] and extensions presented in [2] we show that the median structure of the set of intervals is an algebraic framework supporting the representation of proper, improper and modal intervals. Moreover, we show how arithmetic, containment and relational operations between intervals are formulated in this context without contradicting the initial definitions. Finally, we discuss some theoretical issues and potential applications.

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Verified Solution of an Optimal Control Problem for Elastic Rod Motions Based on the Ritz Method

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Keywords: Optimal Control, Elastic Structure, Verified Solution

Introduction

An important area in control theory remains the optimization of dynamic systems. To model vibrations effectively in flexible structures, generalized variational formulations of PDE control problems were proposed by using the method of integro-differential relations (MIDR) [1]. The comparison of an approximate system and the original model with distributed parameters can be performed explicitly following the MIDR. This approach allows us to estimate the quality of finite-dimensional modelling for refining or coarsening the obtained approximations and, if necessary, to correct the related control law.

The corresponding procedures for solving optimal control problems in linear elasticity based on the Ritz method and finite element technique (FEM) were developed in [2] for uniform elastic rods. The following results extend this approach to the case of nonuniform structures. The novelty of this paper also consists in proving the optimality of some specific motions for the uniform rod, developing an original FEM solver for systems with piecewise constant mechanical parameters, and regularizing the control signals with the help of a quadratic cost functional which includes the discrepancy of constitutive relations.

Problem Statement and Main Results

A generalized formulation of optimal control problems for longitudinal motions of an elastic rod is studied. The only control input is a force applied at one end of the rod. Based on the Ritz method, a finite element algorithm is developed with piecewise polynomial approximations in the space-time domain for unknown displacement, momentum, and force fields. The control strategy aims to depress elastic vibrations in the rod. This is attained by minimizing both mean and terminal energies of the structure over a fixed time horizon.

The results of numerical simulations for the case of nonuniform distribution of mechanical parameters are presented. The verification of optimal control laws has been performed taking into account the explicit local and integral error estimates obtained in accordance with the MIDR. As shown by calculations, the accuracy of approximate solution can dramatically fall down with control optimization. To regulate the appearing error, an upper limit for the quadratic functional of constitutive relations, weakened in the considered variational formulation, is kept by an additional isoperimetric condition.

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Computer-Assisted Existence Proofs for One-Dimensional Schrödinger-Poisson Systems

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Keywords: Computer-Assisted Proof, Schrödinger-Poisson, Existence

We are aiming at non-trivial solutions of the one-dimensional time-independent Schrödinger-Poisson system

$$\left.\begin{aligned} -u'' + Vu + \Phi_u u &= f(u) \\ -\Phi_u'' + c\Phi_u &= u^2 \end{aligned}\right\} \text{ on } \mathbb{R},$$
$$\lim_{x \to \pm\infty} \Phi_u = 0$$

where $c > 0$ is an additional parameter needed in the one-dimensional case, $V \in L^\infty(\mathbb{R})$ is a positive potential and $f \in C^1(\mathbb{R})$.

This one-dimensional system is a simplified model for the three-dimensional time-dependent version

$$\left.\begin{aligned} -i\hbar\partial_t\psi - \frac{\hbar^2}{2m}\Delta\psi + q_e W_e\psi &= f(\psi) \\ -\varepsilon\Delta W_e &= q_e|\psi|^2 \end{aligned}\right\} \text{ on } [0, \infty) \times \mathbb{R}^3$$
$$\lim_{|x| \to \infty} W_e = 0$$

which (for $f \equiv 0$) plays an important role in today's semiconductor technology. ψ represents the wavefunction of a particle, in our case of an electron, and m is its mass. W_e describes the electric potential which depends on the wavefunction ψ by the above Poisson equation with Dirichlet boundary conditions.

To prove non-trivial solutions of the one-dimensional Schrödinger-Poisson system we first "solve" the second equation using the corresponding Green's function Γ, and insert the result into the first one:

$$-u'' + \left(V + \int_{-\infty}^{\infty} \Gamma(\cdot, t) u(t)^2 \, dt \right) u = f(u) \text{ on } \mathbb{R}.$$

Furthermore u should be a solution with finite energy level for physical reasons, i.e. we look for a solution $u \in \mathrm{H}^1(\mathbb{R})$.

Applying computer-assistance to the above equation, we are able to prove the existence of a non-trivial solution of the one-dimensional Schrödinger-Poisson system for the case $c = 50$, constant potential $V \equiv 1$, and the nonlinearity f chosen as $f(y) = y^3$ ($y \in \mathbb{R}$).

Starting from a numerical approximate solution, we compute a bound for its defect, and a norm bound for the inverse of the linearization at the approximate solution. For the latter, eigenvalue bounds play a crucial role, especially the eigenvalues "close to" zero. Therefor we use the Rayleigh-Ritz method and a corollary of the Temple-Lehmann theorem to get enclosures of the eigenvalues of the linearization below the essential spectrum.

With these data in hand, we can use a fixed-point argument to obtain the desired existence of a non-trivial solution "nearby" the approximate one. In addition to the pure existence result, the used methods also provide an enclosure of the exact solution.

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Traveling Waves in a Chemotaxis Model

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Keywords: PDEs, Chemotaxis, Reaction-Diffusion, Traveling Waves, Computer Assistance

Introduction

In this talk we dive into the subject of dynamical systems and in particular a dynamical system that mathematically models a natural phenomenon called **Chemotaxis**.

One research area in which Chemotaxis is heavily involved is cancer research. In particular, a lot of time and effort are invested in the understanding of Metastasis (i.e., the final stage of cancer) but to understand those complicated processes one has to understand the mechanism behind them, i.e., Chemotaxis.

The Model

The classical mathematical model for chemosensitive movement is the Patlak-Keller-Segel model [2, 3], a hyperbolic-parabolic reaction-diffusion system of PDEs which assumes diffusion of the species at hand. In our approach we consider an alternative system of PDEs derived by Dolak and Hillen [1] that applies **Cattaneo's law of Heat propagation** with finite speed i.e.

$$
\begin{aligned}
u_t + q_x &= F(u) \\
c_1 q_t + q &= -c_2 u_x - V(u, S) S_x \\
S_t &= c_3 S_{xx} + G(u, S)
\end{aligned}
\tag{1}
$$

where subscripts x, t denote the partial derivatives $\frac{\partial}{\partial x}, \frac{\partial}{\partial t}$ respectively and c_1, c_2, c_3 are known constants.

Aims, Approach, Techniques

Our aim is to rigorously prove existence of special solutions called **traveling waves** (TW) on an unbounded domain $\Omega = \mathbb{R}$ with **computer assistance** wherever this is needed.

We approach the problem as follows: substituting in (1) the traveling wave ansatz $u(x,t) = w(x - \mu t)$ and considering appropriate boundary conditions we write our problem in the form

$$F(w, \mu) = 0$$

where $F : \mathbf{X} = H^1(\mathbb{R})^4 \times \mathbb{R} \to L^2(\mathbb{R})^4 \times \mathbb{R} = \mathbf{Y}$, $w \in H^1(\mathbb{R}^4)$ is the TW and μ its speed.

To prove existence of the TW we follow a method proposed by Plum [4] from an elliptic PDE problem and adapt them to our hyperbolic-parabolic system. The techniques of this proof involve the computation of defect bounds, eigenvalue inclusion methods, spectral bounds and Banach's Fixed Point theorem analytically and into interval arithmetics environment.

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Reliable Visual Analytics as Part of a Process Quality Assessment

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Keywords: Reliable Visual Analytics, Verification and Validation
Analysis

With the omnipresent advance of computing, the Internet of Things
or cloud based technologies, ambient intelligence and smart environ-
ment software gain more and more importance for supporting mobile
users in all areas of their daily life. Typical challenges along this way
are huge amounts of heterogeneous input and output data and high
system complexity, for dealing with which new visual and collabora-
tive approaches are called for. The emerging area of visual analytics
(VA) offers a solution to these problems, its main strength lying in
the ability to engage in the analytical process the whole of human
perceptual and cognitive capabilities augmented by advanced compu-
tations [1]. VA hardware and software architectures serve to assess
and visualize important system/process parameters, descriptors and
uncertain environment entities. Only few publications explicitly use
the term Reliable Visual Analytics or propose guidelines for assess-
ment of VA frameworks, methodologies, applicability and efficiency.
Several authors focus on device-dependent transformation, accurate
understanding of outcome using reliable mapping algorithms and stan-
dardized procedures to automatically select, analyze, refine and com-
bine visual data. Sometimes accuracy and reliability are explicitly or

implicitly addressed in the context of uncertain data acquisition, aircraft and power plant safety, risk assessment and healthcare monitoring and management. This means, on the one hand, that human-centered paradigms become an important feature within a workflow for designing, modeling, and implementing various real life processes [2]. On the other hand, reliability of VA architectures have to be ensured to apply them in the context of formal verification and validation (V&V) assessment. Developing reliable VA frameworks needs guidelines and recommendations based on real world use cases, benchmarks, lab studies. Further generic requirements might concern ethical considerations or interaction and collaboration styles, for example, via virtual reality 3D devices [3]. In this contribution, we assess the strengths and weaknesses of big data and VA science and technology in the context of V&V. This includes various collaboration methodologies and mixed reality platforms where scientists of different disciplines interact with each other, with data and with information. We discuss the possibility of a multilayer quality assessment procedure similar to that from data analytics, bearing in mind the methodologies from the neighbouring fields concerning reliability, accuracy, performance efficiency, group activity monitoring as well as validation and evaluation. Further topics include appropriate hardware devices managing, for example, safe data transfer, and the contributing concepts from cognitive and perceptual sciences.

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Interval Computations in C++20

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Summary of the Talk

The speaker was introduced to C++ in the early 1990s, by reading the books by Lippman [1] and Barton & Nackman [2], and has used this language in several academic and industrial projects since that time. The programming world has changed considerably in this period, but he still uses C++ for most of his scientific computations because, as more recent languages like R, Python and Julia, C++ is also evolving and is supported by a very active community. Moreover, C++'s latest versions have several features which are quite useful for interval arithmetic and scientific computing in general, both in terms of performance and usability.

This talk describes how modern C++ can be used for interval computations, in practical examples. These examples use the Moore library for interval arithmetic [3], and its extensions for linear algebra, automatic differentiation and Taylor models. We would like to show that with C++ we can write code for interval computations which is intuitive, general and efficient. By "intuitive" we mean code like this:

```
interval<> x(1,2), y(3,4);
std::cout << x * y + exp(x + cos(y));
```

and by general we mean code like the next snapshot, in which T could be, for instance, a floating point number with a mantissa of 1024 bits.

```
interval<T> x("[1,2/325]"), y("[3/431,4/25]");
```

We could also use matrices and vectors efficiently and intuitively. For instance in the code below we create vectors and matrices with interval entries (`ivectors` and `imatrices`) fill the large ones with random intervals (irand) and perform some arithmetic operations.

```
ivector<3>   small_vec{  i(1,2), i(2,3), i(3,4) };
imatrix<2,3> small_mat{ {i(0,1), i(2,3), i(0,2)},
                        {i(0,1), i(0,2), i(0,2)} };

ivector<> large_vec(250);
imatrix<> large_mat(2,250);
generate(large_vec, irand);
generate(large_mat, irand);
auto prod = small_mat * small_vec + large_mat * large_vec;
```

Finally, C++ leads to efficient code because template meta programming allows us to perform part of the computation at compile time, and give more optimizing opportunities for the compiler. For instance, the small vector and matrix above are stored on the stack, and their sizes are known at compile time, whereas the large vector and matrix are allocated on the heap. The compiler deduces that `prod` is a two dimensional vector, which is allocated on the stack, and there is no heap allocation while evaluating it: the compiler knows that the intermediate results in the computation of `prod` are small two dimensional vectors, and their evaluation involves only the stack, using expression template techniques.

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Safe Approximation of Probabilities

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Abstract

Interval methods can be seen as providing a tool to reason about deterministic systems with imprecise inputs. This views can be justified formally through the notion of set extension of functions, and the fact that interval methods "arithmetize" certain types of set computation. A natural question to ask is whether similar view is possible when we wish to move from sets to distributions. A related question is whether we can generalize this idea for deterministic systems to non-deterministic (set-valued) and probabilistic (distribution valued) systems. This work presents a mathematical framework for answering these and related questions. The framework makes extensive use of two notions that have found extensive use in programming language semantics, namely, monads and monad transformers.

Keywords: Probabilities, Approximation, Intervals, Monads

Introduction

The design of high confidence systems often relies on the computation of probabilities that may, for instance, give an upper bound to the probability of failure. Traditional numerical methods are too error prone for such applications. In contrast, interval methods can deliver safe approximations in the form of an enclosing set (rather than some near by point) approximations of points. We propose a semantic framework for using such methods for probabilistic models. Such semantics may provide a reference model to establish correctness of interval algorithms.

The proposed framework builds on two key observations, namely, that (discrete) probability distributions form a monad on sets, and that the non-empty powerset monad can be turned into a monad transformer (unfortunately, this result relies on the axiom of choice). This transformer allows us to add non-determinism to any monad on sets, including probability distributions.

The framework thus generalizes the notion of set-extension, which provides a basis for rigorous interval computation.

Monads

We begin by recalling the definition of the notion of a monad:

Def 1 (Monad, c.f. Moggi [1]). *A monad on the category* Set *of sets is a triple* $(M, \eta, \_^*)$ *such that if* X:Set *then* MX:Set, η:$X \to MX$ *is a map (from* X *to* MX), *if* f:$X \to MY$ *then* f^*:$MX \to MY$. *Furthermore,* η *and* f^* *must satisfy the following equational axioms for any* f:$X \to MY$ *and* g:$Y \to MZ$

1. $\eta_X^* = id_{MX}$

2. $(g^* \circ f)^* = g^* \circ f^*$

3. $f^* \circ \eta_X = f$

Trivial examples of monads are the identity $MX = X$ and the terminal monad $MX = 1$, where 1 is a singleton set. For this work the most interesting monads are:

- Exceptions $X + E$, where $+$ denotes disjoint union (η and f^* should be obvious).

- Powersets (non-determinism) $P(X)$, where $P(X)$ is the set of subsets of X, $\eta(x) = \{x\}$ and $f^*(A) = \bigcup_{x:A} f(x)$; also $P_+(X)$, i.e., $P(X)$ without the empty set, is a monad.

- Probability Distributions $D(X) = \{p:X \to [0,1]| \sum_{x:X} p(x) = 1\}$, $\eta(x)(x') = 1$ if $x = x'$ else 0 and $f^*(p)(y) = \sum_{x:X} p(x) * f(x)(y)$.

  An equivalent monad is given by the set $D'(X)$ of measures, i.e., $\mu:P(X) \to [0,1]$ such that $\mu(X) = 1$ and $\mu(\uplus_{i:I} A_i) = \sum_{i:I} \mu(A_i)$ for any family $(A_i|i:I)$ of disjoint subsets of X. The correspondence between $D(X)$ and $D'(X)$ is $\mu(A) = \sum_{x:A} p(x)$ and $p(x) = \mu(\{x\})$.

Other examples can be found in the reference above, subsequent work in the context of programming languages semantics and in the design of functional programming languages with IO and computational effects.

Interval for Probability Distributions

The natural order on $[0,1]$ induces a point-wise order on the function space $X \to [0,1]$. This allows to introduce interval notations for subsets of probability distributions in $D(X)$, for instance

$$[l, u] = \{p:D(X)|\forall x.l(x) \le p(x) \le u(x)\}$$

where $l, u:X \to [0,1]$ (not necessarily in $D(X)$). Another notation is

$$[L, U] = \{p:D(X)|\forall(A, l_A):L.l_A \le p(A) \land \forall(B, u_B):U.p(B) \le u_B\}$$

where $L, U:(P(X), [0,1])^*$ are finite sequences and p is extended additively to subsets of X, namely $p(A) = \sum_{x:A} p(x)$.

M-Extensions of Functions

The natural set-extension of $f:X \to Y$ is the map $P(f):P(X) \to P(Y)$ such that $P(f)(A) = \{f(x)|x:A\}$. This maps satisfies the equation $P(f)(\{x\}) = \{f(x)\}$, which establishes that it is an extension of f. The notion of extension generalizes to any monad, and hinges on the fact that a monad is also a functor.

Def 2 (Functor). *A functor F on* Set *maps a set X to a set $F(X)$, and $f:X \to Y$ to $F(f):F(X) \to F(Y)$ so that*

1. $F(id_X) = id_{F(X)}$

2. $F(g \circ f) = F(g) \circ F(f)$

Def 3 (Natural Transformation). *A natural transformation τ from a functor F to a functor G is a family of maps $\tau_X : F(X) \to G(X)$ indexed by X:Set such that for any $f : X \to Y$ we have $\tau_Y \circ F(f) = G(f) \circ \tau_X$.*

Prop 1 (M-extension). *A monad becomes a functor by defining $M(f) = (\eta_Y \circ f)^*$ for $f : X \to Y$, and $\eta_X : X \to M(X)$ becomes a natural transformation from the identity functor to M, i.e., $(Mf)(\eta_X(x)) = \eta_Y(f(x))$.*

When M is the powerset monad P, one recovers as a special case the natural set-extension. For almost every monad on Set the map η_X is injective, thus one can view X as a subset of $M(X)$ and $M(f)$ as an extension of f.

Approximating Distributions

We now present an application of the framework above, namely, safely approximating the distribution-extension of a given function.

More precisely, given an approximation F of a function $f : X \to Y$ and a lower approximation L of a distribution $\mu : D'(X)$, we want to compute an approximation $[L', U']$ of $\mu' = D'(f)(\mu) : D'(Y)$.

To define the algorithm that solves this problem we must first specify the type of approximations involved and the properties that they must satisfy. Recall that the set-extension of $f : X \to Y$ is the map $P(f) : P(X) \to P(Y)$ such that $P(f)(A) = \{f(x)|x{:}A\}$, and that $\mu' = D'(f)(\mu)$ means that $\mu'(B) = \mu(f^{-1}(B))$.

Inputs: An approximation F of f, namely a map $F : P(X) \to P(Y)$ such that $\forall A{:}P(X).P(f)(A) \subseteq F(A)$.

A lower approximation $L = [(A_i, l_i)|i{:}n]$ of μ, i.e., $\forall i{:}n.l_i \leq \mu(A_i)$, with $(A_i|i{:}n)$ partition of X (thus $\sum_{i:n} l_i \leq 1$).

Output: An approximation $[L', U']$ of $\mu' = (D'f)(\mu)$, namely two sequences $L', U':(P(Y) \times [0,1])^*$ such that $\forall(B', l'):L'.l' \le \mu'(B')$ and $\forall(B', u'):U'.\mu'(B') \le u'$.

Algorithm: For convenience, we identify a natural number n with the set $\{i|i < n\}$ of its predecessors. The algorithm proceeds as follows:

1. For $I \subseteq n$, let $A_I = \uplus_{i:I}A_i$, $l_I = \sum_{i:I} l_i$ and $u_I = 1 - l_{I^c}$, where $I^c \subseteq n - I$ is the complement of I.

   Note: $l_I \le \mu(A_I) \le u_I$ holds by the assumption on L, in other words from the lower approximation L we compute its *completion* $[L^\sigma, U^\sigma]$, where $L^\sigma = [(A_I, l_I)|I \subseteq n]$ and $U^\sigma = [(A_I, u_I)|I \subseteq n]$, which approximates the same probability distributions, but more explicitly.

2. Let $B_I = F(A_I)$.

   Note: Since $f(A_I) \subseteq F(A_I) = B_I$ we have $A_I \subseteq f^{-1}(B_I)$. Thus, $l_I \le \mu'(B_I)$. Furthermore, $\mu'(B_I^c) \le u_{I^c}$, as $f^{-1}(B_I^c) = (f^{-1}(B_I))^c \subseteq A_I^c = A_{I^c}$.

3. $L' = [(B_I, l_I)|I \subseteq n]$ and $U' = [(B_I^c, u_{I^c})|I \subseteq n]$.

Monad Transformers

Functors compose, while monads may not. More precisely, if M and M' are monads, then $M' \circ M$ is a functor, $\eta'_{MX} \circ \eta_X:X \to M'(MX)$ is a natural transformation, but there is no canonical way to define f^* for $f:X \to M'(MY)$.

Prop 2. *If M is a monad, then $M(+E)$ and $P_+(M())$ are monads.*

Hint   Given $F:X \to P_+(MY)$, let $\Pi x:X.F(x)$ be the set of *choice maps* f st $\forall x:X.f(x):F(x)$, then $F^*(A) = \{f^*(c)|c:A \wedge f:\Pi x:X.F(x)\}$. Proving that F^* satisfies the axioms for monads, use crucially the axiom of choice.

Monad morphisms are a way to relate two monads, more precisely they relate monads in the *same way* as natural transformations relate functors, thus one can define a category of monads on Set. Since monads may not compose, monad transformers are a way to build incrementally complex monads from simpler ones.

Def 4 (Monad Morphism). *A monad morphism is a natural transformation σ from a monad M to a monad M' such that :*

$$\eta'_X(x) = \sigma_X(\eta_X(x)) \qquad (\sigma_Y \circ f)^{*'}(\sigma_X(c)) = \sigma_Y(f^*(c))$$

We write Mon for the category of monads and monad morphisms.

Def 5 (Monad Transformer). *A monad transformer consists of a functor T on Mon and a natural transformation $\eta^T_M:M \to T(M)$ from the identity functor on Mon to T.*

Unlike monads, monad transformers can be composed. Prop 2 can be turned into two examples of monad transformers:

- Let $\iota_X:X \to X + E$ and $\iota'_X:E \to X + E$ the obvious inclusion maps, the monad transformer T adding exceptions is given by $(TM)(X) = M'X$, where M' is the monad $M'X = M(X + E)$, $\eta'_X = \eta_{X+E} \circ \iota_X$, if $f:X \to M'Y$ then $f^{*'} = g^*$ with $g:X+E \to M'Y$ such that $g(\iota(x)) = f(x)$ and $g(\iota'(e)) = \eta_{Y+E}(\iota'(e))$.

  If $\sigma:M \to M'$ is a monad morphism then $(T\sigma)_X = \sigma_{X+E}$, and $\eta^T_M:M \to M'$ is the monad morphism such that $\eta^T_{M,X} = M(\iota_X)$.

- The monad transformer T adding non-determinism is given by $(TM)(X) = M'X$, where M' is the monad $M'X = P_+(MX)$, $\eta'_X(x) = \{\eta_X(x)\}$, if $F:X \to M'Y$ then $F^{*'}(A) = \{f^*(c)|c:A \land f:\Pi x:X.F(x)\}$.

  If $\sigma:M \to M'$ is a monad morphism then $(T\sigma)_X = P_+(\sigma_X)$, and $\eta^T_M:M \to M'$ is the monad morphism such that $\eta^T_{M,X}(c) = \{c\}$.

Other examples of monad transformers are $(TM)(X) = (MX)^S$ and $(TM)(X) = M(X \times S)^S$, the latter is not given by functor composition.

Related Work

Weichselberger [2] (Def 2.2) introduces R-probabilities, namely a pair of maps L and U from a σ-algebra (called σ-field in [2]) $\mathcal{A}$ on a sample space Ω, which bound the probability distributions on Ω, namely $\forall A{:}\mathcal{A}.L(A) \leq p(A) \leq U(A)$. In this paper we work in a simplified setting, i.e.: the space Ω is a set X, the σ-algebra $\mathcal{A}$ is the powerset $P(X)$, L and U are finite sequences $L = [(A_i, l_i)|i{:}m]$ and $U = [(B_j, u_j)|j{:}n]$ representing maps $L', U'{:}P(X) \to [0,1]$, namely $L'(A) = l_i$ when $A = A_i$ otherwise 0, and $U'(B) = u_j$ when $B = B_j$ otherwise 1. An algorithm should manipulate the finite list L and U, while its correctness should be expressed in terms of the maps L' and U'.

We have not gone beyond probability distributions on sets. For distributions on spaces, the approach taken in [5] would be useful.

Future Work

In this work we have focused on extensions of functions. In general, we may wish to extend systems that are richer than simply functions. A *transition system* (TS) is a pair (S, R) with R a binary relation on the set S. There is a bijection between $R{:}P(S^2)$ and $t{:}S \to P(S)$. This suggests a generalization of TS by replacing P with a monad M.

Def 6 (Monadic Transition System). *Given a monad M on* Set, *an M-TS is a map $t{:}S \to M(S)$, and moreover the map t allows to define the following auxiliary maps*

- $T{:}\Pi \to (MS \to MS)$, *where $T_0(c) = c$ and $T_{n+1}(c) = t^*(T_n(c))$, gives the* configuration *reached in n steps when starting from c;*

- $T'{:}\Pi n{:}\mathbb{N}.MS \to M(S^{n+1})$ *gives the* traces *of length n starting in c, namely $T'_0(c) = c$ and $T'_{n+1}(c) = f^*(T'_n(c))$, with $f = (l, s){:}S^{n+1} \mapsto g^*_{l,s}(t(s)){:}M(S^{n+2})$ and $g_{l,s} = s'{:}S \mapsto \eta(l, s, s'){:}M(S^{n+2})$.*

Relevant examples of M-TS for a monad M are

- probabilistic TS, with M the probability distributions monad D

- probabilistic and non-deterministic TS with *trap states*, with M the monad $P_+(D(+E))$, where E is the set of trap states.

A further generalization of transition systems is a map $t{:}S \to B(S)$ with B a functor, in this case t is called a co-algebra for B (c.f. Turi Plotkin [4]). A co-algebra for $B(X) = P(A \times X)$ corresponds to a label transition system (with labels in A), this B is a functor, but not a monad. However, the definition of the auxiliary maps T and T' rely crucially of having a monad.

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Statistics for Imprecise Data

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Introduction

In every branch of science and engineering, the ability to process large volumes of data has become essential. Over the past century, statistics has focused on developing methods for the analysis of datasets with limited sample size. But not all uncertainty in data has to do with small sample sizes.

Imprecise Data

Regardless of the instrument quality each individual datum has always a finite representation. Substituting single measurements with real numbers can be a strong assumption.

Large datasets are often manipulated to produce descriptive statistics, such as mean, median, variance, etc. as well as inferential statistics like regression, classification, outliers detection, etc.

When the data set has imprecision, computing statistics can be challenging. For example, for data in the form of intervals, using naïve interval analysis yields results with inflated uncertainty because of the repeating variables problem. Finding optimal bounds on many statistics is an NP-hard problem that complicates with the size of the data set. It is practically impossible to solve these problems for large data sets with simple space-filling strategies, in which, for instance, the formula for the variance is treated like a black box evaluated for many

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possible configurations of the data points within their respective intervals.

Interval Statistics

In general, computing statistics on interval data sets is at most NP-hard. For example, solving for the lower bound on the variance could be achieved using a quadratic programming algorithm, as the problem can be expressed in the form of a bounded quadratic optimisation, where the decision variables are represented by the data and therefore the constraints are given by the bounds on the interval data. Quadratic programming problems can be solved in polynomial time when linearly constrained and in linear time when not constrained at all, while they are NP-hard in the general case. When the matrix of coefficients of the quadratic function – obtained from the variance arithmetic formula – is positive definite, the lower bound of the variance can be obtained in polynomial time. Sadly, the same is not true for the upper bound, which relates to a concave problem.

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Measurement Errors in Magnetic Resonance Imaging of Fluid Flows

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Keywords: MRI, Flow Measurements, Error Estimation

Introduction

Magnetic resonance imaging (MRI) is commonly associated with medical examinations. It provides a three-dimensional insight into complex structures without requiring optical or physical access. In the past decade, MRI has found increasing application in the field of fluid mechanics (Elkins & Alley 2007). MRI has been used to acquire various flow properties, such as velocity and species concentration in technical fluid systems. In these laboratory experiments, MRI can produce exceptionally high signal-to-noise ratios, high resolution and sharp contrasts compared to medical imaging.

As for any other measurement technique, it is essential to estimate the statistical uncertainty of the results. In velocity-sensitive MRI, the uncertainty of the velocity data can be estimated from the distribution of the complex values in the image (Bruschewski et al. 2016). The image is obtained via Fourier Transform from the received signal which is corrupted by thermal noise of the receiver chain and external sources of noise.

In addition to uncertainty estimation, quantitative flow imaging with MRI requires knowledge on all systematic measurement errors that can corrupt the data. In routine flow measurements it is often observed that the fluid velocity leads to errors in the measured geometry. Because the encoding process in common MRI techniques is not instantaneous, the spatial coordinates are encoded at different times. As

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a result, the reconstructed signal of the fluid flow appears at locations that the fluid particles have never physically occupied. An example of this effect is shown in Fig. 1.

The presentation at the meeting will focus on the measurement errors in MRI that are most important when measuring fluid flows. The modeling and estimation of these errors will be discussed and ways will be provided on how to avoid these errors.
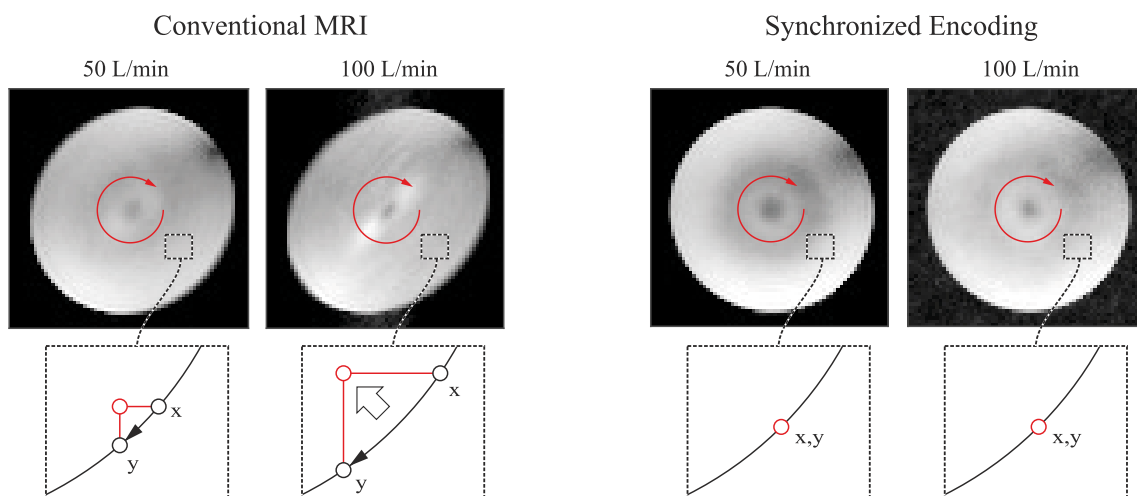
Figure 1: Image magnitude of a rotating flow measured with a conventional MRI sequence and with a new sequence with synchronized encoding. Because the encoding in conventional MRI is not instantaneous, the spatial coordinates in the flow are encoded at different locations in the fluid leading to distortions in the measured geometry.

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Reachability Analysis for a Class of Uncertain Discrete-Time Systems

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Introduction

A deterministic method to characterize the reachable set of uncertain discrete-time systems is introduced in this work. The class of the considered systems is described by:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{f}(\mathbf{x}_k, \mathbf{w}_k) + \mathbf{B}\mathbf{u}_k \tag{1}$$

where $\mathbf{x}_k \in \mathcal{D} \subset \mathbb{R}^n$ is the state vector and $\mathbf{u}_k \in \mathcal{U} \subset \mathbb{R}^m$ is the input vector. The nonlinear term $\mathbf{f}(.,.)$ stands for the poorly-known part of the system (1), which is assumed to be bounded:

$$\forall \mathbf{x}_k \in \mathcal{D} \text{ and } \forall \mathbf{w}_k \in \mathcal{W} \subset \mathbb{R}^p, \quad \mathbf{f}(\mathbf{x}_k, \mathbf{w}_k) \in [\underline{\mathbf{f}}, \overline{\mathbf{f}}] \tag{2}$$

where $\underline{\mathbf{f}}$ and $\overline{\mathbf{f}}$ are the end-points of the smallest box which contains the range of the function $\mathbf{f}(.,.)$.

Definitions

Before stating the novelty of this study, some definitions about the reachable set of dynamical systems are recalled.

Definition 1. *The reachable set of the uncertain system* (1)*, denoted by* $\mathcal{R}\big([t_0, t_k], t_0, \mathcal{X}_0\big)$*, is the set of all the possible state trajectories generated from an initial set* $\mathcal{X}_0 \subset \mathcal{D}$ *and solutions to the set of difference equations* (1)*.*

Definition 2. *An outer approximation of the reachable set of* (1), *denoted by* $\mathcal{Y}\big([t_0, t_k], t_0, \mathcal{Y}_0\big)$, *is a set that satisfies the following inclusion:*

$$\forall k, \quad \mathcal{R}\big([t_0, t_k], t_0, \mathcal{X}_0\big) \subseteq \mathcal{Y}\big([t_0, t_k], t_0, \mathcal{Y}_0\big) \tag{3}$$

Main Results

The results of this work are mainly inspired from the interval-based state estimation methods presented in [1] and [2]. The following proposition introduces an interval-based predictor to characterize in a deterministic way the reachable set of (1).

Proposition 1. *The interval predictor* (4) *provides an outer approximation* (5) *of the reachable set of the uncertain system* (1).

$$\begin{array}{rcl}
[\mathbf{x}_k] & = & \mathbf{A}^k[\mathbf{x}_0] + [\mathbf{f}_{k-1}] + \mathbf{b}_{k-1} \\
[\mathbf{f}_k] & = & \mathbf{A}^k[\mathbf{f}_0] + [\mathbf{f}_{k-1}] \\
\mathbf{b}_k & = & \mathbf{A}\mathbf{b}_{k-1} + \mathbf{B}\mathbf{u}_k
\end{array} \tag{4}$$

where $\mathbf{b}_0 = \mathbf{B}\mathbf{u}_0$ *and* $[\mathbf{f}_0] = [\underline{\mathbf{f}}, \overline{\mathbf{f}}]$.

$$\mathcal{Y}\big([t_0, t_k], t_0, [\mathbf{x}_0]\big) = \bigcup_0^k [\mathbf{x}_k] \supseteq \mathcal{R}\big([t_0, t_k], t_0, \mathcal{X}_0\big) \tag{5}$$

Moreover, for Hurwitz matrices $\mathbf{A}$, the volume of this outer approximation convergence towards a constant when t_k tends to infinity.

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Interval-Based Global Optimisation for Geodetic Network Adjustment Procedures

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Introduction

The solution of a non-linear least-squares adjustment of a geodetic network is derived from an optimum of the cost function, which is the weighted sum of square residuals. If this cost function shows a multivariate behaviour, it cannot be guaranteed that the solution is a global optimum. Depending on the problem and the initial values for the unknowns, the non-linear adjustment can yield a local optimum only or even fail.

The global optimum is also computable by deterministic optimisation methods for instance by using interval analysis [2-5]. In contrast to probabilistic optimisation methods like simulated annealing or genetic algorithms, the global optimisation based on interval analysis theoretically guarantees to find global optimum in a given interval box presupposed that at least one optimum exists in the interval box [1].

Basic Properties

The search of a global optimum can be performed by a branch-and-bound-strategy as proposed in [6]. In this method a multi-dimensional interval box is defined for the unknowns which have to be optimised. In

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an iterative process the properties of the cost function in this interval box are investigated as presented in [1, 4, 7]. The process is terminated when the global optimum is found with a given accuracy or when it is proven that no optimum exists.

Main Results

This approach is applied to geodetic network adjustment problems. These examples show that a global optimum is found using this method whereby a common non-linear least-squares adjustment (e.g. gradient-based Gauss-Markov model) leads to an incorrect solution. The results from the analysis as well as a discussion of the pros and cons of the proposed approach w.r.t. network adjustment will be presented.

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Guaranteed Bounding Zones for GNSS Positioning by Geometrical Constraints

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Keywords: Intervals, Bounding Methods, Integrity, GNSS

Introduction

Guaranteeing integrity measures is one of the most difficult and important tasks in GNSS positioning. Integrity measures the trust of the navigation information. The user should be alert in time if an error in the navigation system surpasses the given alert limit. The traditional approaches such as least squares (LS) estimation do not guarantee the position solution and outlier detection is based on statistical tests. Interval based methods could be an alternative, where the solution is a set of different shapes e.g. computed by SIVIA or linear programming [1]. In this work we provide a comparison analysis between LS and a Primal-Dual Polytope (PDP) bounding method for GNSS positioning. Primal-Dual method is explained in [2].

Methodology

In both methods, the GNSS navigation Eq. (1), has to be linearized via Taylor expansion at approximate initial position [3]. The LS solution is shown in Eq. (2), and the constraints equation of the PDP algorithm in Eq. (3) [1], where sv identifies the space vehicle and ur the user, ρ the pseudorange measurements, cdt the clock offset, $\mathbf{A}$ the

design matrix, $\mathbf{P}$ the weight matrix, $\mathbf{d}\rho$ the observed minus computed, $\mathbf{d}\hat{\mathbf{x}}$ the estimated state vector, and Δ the interval error bound of the observations. LS estimation which represented in Eq. (2), is the estimated correction to be applied on initial position. If the initial position is not known, iterative LS estimation has to be applied. LS is totally based on stochastic models where a confidence ellipsoid can be derived from the variance covariance matrix $(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}$, and outlier detection in this case is purely based on hypothesis statistical test. Instead, PDP method apply a deterministic interval error bound on the measurements and provide a consistency check bounding zone which also reflect the directions of the uncertainties in the position domain. The interval error bound can be derived in different ways as explained in [1] and [4].

$$\rho = \sqrt{(x_{sv} - x_{ur})^2 + (y_{sv} - y_{ur})^2 + (z_{sv} - z_{ur})^2} + cdt_{sv-ur} \qquad (1)$$

$$\mathbf{d}\hat{\mathbf{x}} = ((\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P})\mathbf{d}\rho \qquad (2)$$

$$\mathbf{d}\rho - \Delta \leq \mathbf{A}\mathbf{d}\hat{\mathbf{x}} \leq \mathbf{d}\rho + \Delta \qquad (3)$$

The PDP algorithm converts a convex set of constraint inequalities Eq. (3), which represent a H-polytope into a set of vertices at the intersection of those inequalities which represent a V-polytope.

Simulated scenarios and real test drives have been analyzed to gain a good understanding of the GNSS positioning results from both methods. Test data were recorded using Novatel Span system consisting of a dual frequency GNSS receiver equipped with 2 antennas and an iMAR FSAS IMU. Calibration of the platform was done in the Geodetic Institute laboratory Hannover using a Leica absolute laser tracker AT930 with submillimeter accuracy. The test drive is a small set of a measurement campaign which has been conducted by the DFG research training group "I.C.SENS". The ground truth is provided by the post processing software TerraPos where the 2 GNSS antenna were tightly coupled with the IMU measurements.
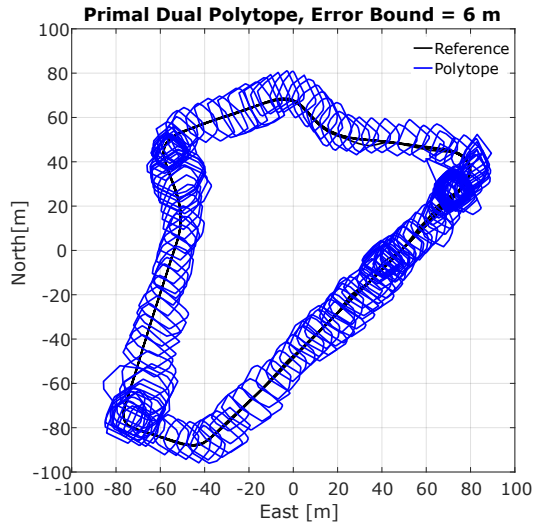
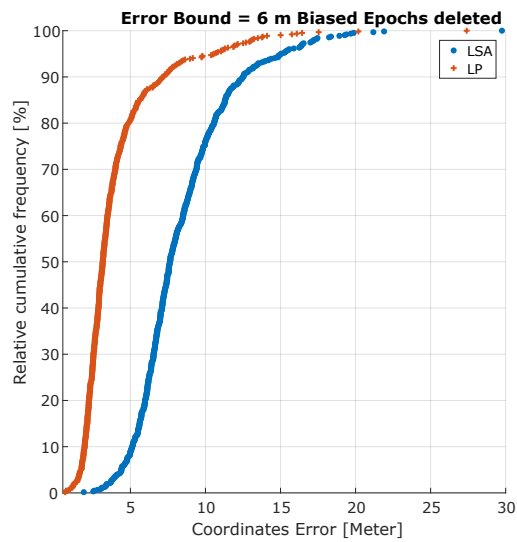Figure 1: PDP solution sets of GPS code measurements.

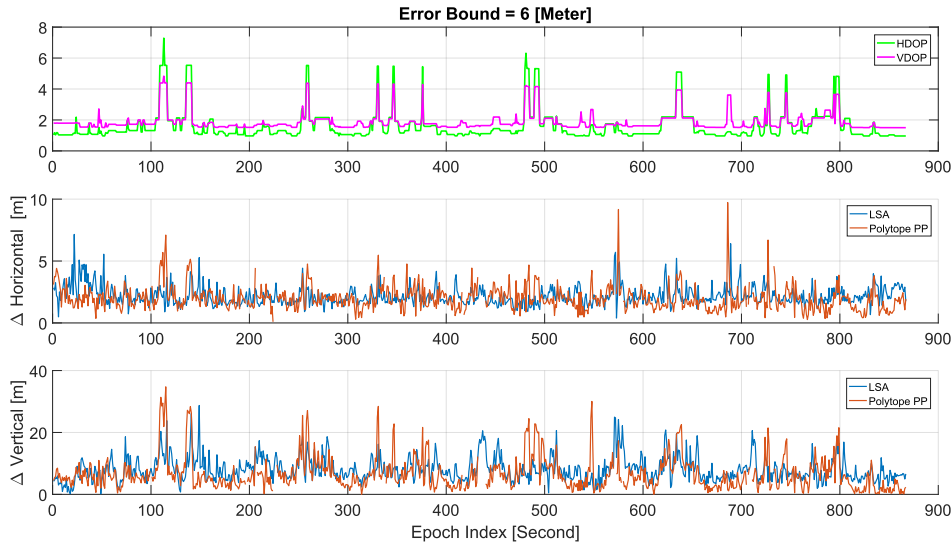Figure 2: Relative cumulative frequency of the coordinate errors.

Figure 3: Point positioning errors in horizontal and vertical directions.

Main Results

Interestingly, the volume and shape of the polytope is an inconsistency measure rather than a confidence measure. It is related to the positioning geometry and revels observation of maximum impact. If a point position (PP) is needed, the barycenter of the polytope can be computed and eventually the variances. Moreover, minimum detectable biases are derived from interval bounds and polytope shapes. Fig. 1, shows polytopes obtained form PDP applied on the GPS code measurements of the first antenna with constant error bounds equal to 6 meters in topocentric coordinate system. It is clear that, all the polytopes contain the true solution. However, some of the measurements are outliers and their set solution is empty (discontinuity in polytope PP errors in Fig. 3), which provides guaranteed bias detection. Fig. 3, shows the errors in horizontal and vertical direction of the point positioning from LS and PDP algorithms. When we have bad geometry (large DOP values), PDP solution deteriorate and shows higher peaks than LS. However, the overall performance of PDP PP shows better results than LS where the relative cumulative frequency of the

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coordinate errors of PDP over-perform LS (see Fig. 2).

As a conclusion, polytope PP is more accurate and precise than the LS PP, but it is more sensitive to the navigation geometry. Moreover, PDP algorithm provides guaranteed bias detection and exclusion.

Acknowledgement

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RGB-Laser Odometry Under Interval Uncertainty for Guaranteed Localization

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Introduction

To navigate, robots have to localize themselves in the environment. Since GPS might not be available, robots need to estimate their ego-motion gradually using different sensors such as cameras, laser scanners and/or inertial measurement units (IMUs). In the past, we developed a probabilistic approach to estimate the robot's odometry using a monocular camera and a depth sensor while simultaneously estimating the sensors' clock offsets [1]. Nevertheless, our approach and most other approaches [2] focus on computing a point-valued position for the robot only while neglecting the uncertainty of the pose estimation. However, due to the imperfections of every sensor, this point position might be erroneous. Further difficulties are the nonlinearity of the problem, which can lead to further deviations from the true position, and outliers, which can affect the result of these approaches since they tend to compute a solution that satisfies all observations. To overcome these issues, we propose an interval-based approach. Our method fuses camera, laser scanner and IMU information while taking the sensors' uncertainties into account to compute intervals for the robot's pose in 3D. Similar work was done by Kenmogne et al. [3], but they compute their robot's position relative to known landmarks, which we assume to be not available. Bethencourt and Jaulin [4] solve a similiar problem to ours, but apply it to 3D reconstruction instead of localization. While

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we do not believe that interval-based approaches will replace existing probabilistic approaches, our approach can be used to constrain the initial search space of probabilistic methods or to detect errors in the probabilistic solution.

Method

We estimate the robot's motion from one image frame to another. First, we find corresponding image features between image frames using SIFT [5]. Since depth (distance) information is needed to estimate the odometry from matched feature points, we use scan points from a laser scanner and find a guaranteed interval for a feature's depth. For this, we assume an unknown but bounded error for both sensors and for the transformation between the sensors. By projecting the laser scanner's scan boxes onto the image plane, we find all possible scan points for an image feature and calculate the depth as the union over all those scan boxes' depths. Our method is keyframe-based, which means that we estimate the motion from the current image frame relative to the most recently defined keyframe until we have to insert a new keyframe (e.g. if we cannot match enough features). To contract the intervals for the motion between two frames, we use a forward-backward contractor based on the rigid body transformation

$$\mathbf{X}_i^k = \mathbf{R}\mathbf{X}_i^c + \mathbf{T}, \tag{1}$$

where $\mathbf{X}_i^k$ and $\mathbf{X}_i^c$ are the 3D coordinates of the same feature i in the keyframe k and the current frame c, respectively. $\mathbf{R}$ and $\mathbf{T}$ are the rotation matrix and the translation vector, for which we want to contract the intervals. By reformulating the equation it is possible to also include features with depth information in one frame only or without any depth information. To find an initial enclosure for the rotation parameters, we use measurements from the IMU. Since some feature matches might be wrong, we use a relaxed intersection for the contractors to not only account for, but also identify outliers. Finally, if we have to insert a new keyframe, we use further constraints to

(a) GPS correction every three seconds.   (b) GPS correction at new keyframe.
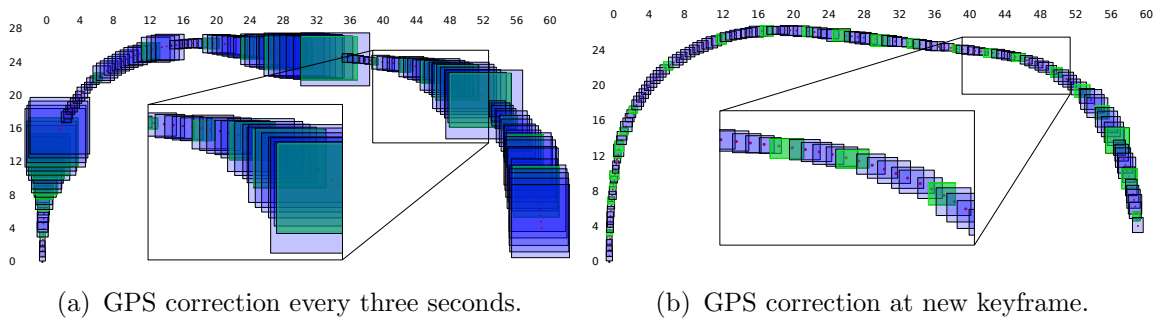
Figure 1: Odometry results.

contract the intervals for the motion to the previous keyframe (like bundle adjustment).

First Results

To evaluate our approach we use small sequences from the KITTI data set [6]. After some time, we use GPS measurements to contract the intervals. Otherwise, the localization uncertainty grows infinitely large due to drift and the boxes convey no information anymore. For future work, we plan to build a map or use loop closure to prevent drift. In the first experiment (c.f. Figure 1(a)) we use GPS measurements every three seconds; in the second experiment (c.f. Figure 1(b)) we use GPS measurements whenever we have to insert a new keyframe. In both figures the red dots depict the true solution (GPS), the blue boxes depict our localization results and the green boxes depict a keyframe. It can be seen that the true solution is always enclosed in our intervals.

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Bringing Interval Methods Into ROS, the Robot Operating System

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Introduction

The Robot Operating System [1] (ROS) is a collection of tools and libraries helpful for robotic applications. It is centered on a message passing middleware, provides data recording, replay and visualization tools, device drivers, and implementations of robotic algorithms (navigation, planning...).

We present the *interval* ROS package. It enables the use and display of interval data in the ROS framework.

Interval Data Types: *interval_msgs*

One of the key features of ROS is message passing between so called *nodes*, in a publish-subscribe pattern. ROS comes with a set of predefined messages, from basic types (like `Bool`, `Float64`, `String`...) to complex types specific to robot applications (like `Point`, `Image`, `Path`...).

The first step to facilitate interval computations with ROS is to define interval message types, so that interval computation nodes can communicate. Intervals are defined for the basic `Float`, `Time` and `Duration` types using two fields: a lower bound (`lb`) and and an upper bound (`ub`). Boxes are defined, both as arrays of the basic interval types, and as the interval counterpart of common messages like `Point` and `Vector3`. We also provide structures representing subpavings.

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Visualization: *interval_rviz_plugins*

RViz is the 3D visualization tool provided with ROS. It features off-the-shelf ways of displaying positions, paths, maps and images, while performing all necessary transformations to convert all data into the same reference frame. The *interval_rviz_plugins* ROS package contains RViz plugins, enabling "click and play" display of boxes and subpavings in the RViz window.

Interval Computations: *interval_tools*

The *interval_tools* package provides utility nodes for conversion between standard ROS point-valued types and interval types: "inflating" standard messages into interval messages, extracting the midpoint of an interval message (interval or subpaving), extracting the radius of a box, etc. This is useful for easy interfacing with existing ROS nodes.

The package also contains a *SIVIA* node, which publishes the result of Set Inversion via Interval Analysis as subpavings [2]. This node uses contractors from the IBEX library [3]. The expression of the function to be inverted is configurable as a parameter.

The processing and visualization capabilities of the *interval* ROS package will be demonstrated with a live set-membership estimation problem.

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Cooperative Localization of Drones by Using Interval Methods

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Keywords: Cooperative Pose Estimation, Set Inversion

Introduction

In this paper, we address the problem of cooperative pose estimation [1] in a group of N unmanned aerial vehicles (UAV), each equipped with a camera that sees landmarks with known positions. The UAVs communicate, exchange poses and measure distances with neighbours and a base station. Our aim is to compute the pose domain of each robot assuming the errors on measurements are bounded.

Single Robot Pose Estimation

Each robot first estimates its pose domain $\mathbf{r} = (x, y, z, \phi, \theta, \psi)$ using camera and base distance constraints. To get the camera constraints, the perspective projection equation (Eq. (1), pinhole camera model) of a 3D world point $^w\mathbf{X}$ in the camera frame represented in normalized coordinates $\mathbf{x} = (^cx, {}^cy)$ is used (see [2] in camera only case).

$$\mathbf{x} = \mathbf{\Pi} \, {}^c\mathbf{T}_r \, {}^r\mathbf{T}_w(\mathbf{r}) \, {}^w\mathbf{X} \tag{1}$$

with $^r\mathbf{T}_w$ the unknown transformation matrix between the world reference frame and a frame attached to the robot and $^c\mathbf{T}_r$ is the known rigid transformation between the camera and the robot frames. Eq. (1)

is applied for each visible landmark $^w\mathbf{X}_i$ $(i \in 1..m)$ and the following constraints can be derived

$$
C_i: \begin{cases}
(^cX_i, {}^cY_i, {}^cZ_i) = {}^c\mathbf{T}_r \, {}^r\mathbf{T}_w(\mathbf{r}) \, {}^w\mathbf{X}_i \\
{}^cx_i = \frac{{}^cX_i}{{}^cZ_i}, {}^c y_i = \frac{{}^cY_i}{{}^cZ_i}, \\
{}^cx_i \in [{}^cx_i], \ {}^cy_i \in [{}^cy_i], {}^cZ_i \ > \ 0.
\end{cases}
\tag{2}
$$

The image/range-based pose estimation problem is then defined as a constraint satisfaction problem (CSP)

$$
\mathcal{H}: \left(\begin{array}{c}
\mathbf{r} \in [\mathbf{r}], \\
\{C_i, \ i \in 1...m\} \\
C_{dist}
\end{array} \right),
$$

where C_{dist} is the additional distance constraint between the robot position $p = (x, y, z)$ and the base station B used to get a tighter pose estimate

$$
C_{dist}: d = \|\mathbf{p} - \mathbf{b}\|_2, \quad d \in [d]
$$

with $\mathbf{b}$ the known position of the base station.

A robot R_k computes a domain in a form of an outer subpaving $\mathbb{S}_{\mathbf{r}_k}^+$, that contains all the feasible poses, using SIVIA [3] to solve $\mathcal{H}$.

Robots Cooperation: Data Exchange

At each time step, once the pose domain $\mathbb{S}_{\mathbf{r}_k}^+$ is computed, the robot exchanges the bounding box of its position domain $[\mathbf{p}_k] = \Box \operatorname{proj}_\mathbf{p} \mathbb{S}_{\mathbf{r}_k}^+$, where $\Box$ is the bounding box operator, and $\operatorname{proj}_\mathbf{p}$ is the projection onto the position space. The position $[\mathbf{p}_k]$ is transmitted to all neighboring robots R_j, $j \in \mathcal{N}(k)$, and the distances $d_{k,j}$ between R_k and R_j are simultaneously measured (with $\mathcal{N}(k)$ the neighbours of R_k).

At reception of information (position boxes $[\mathbf{p}_j]$ and bounded-error distances measurements $[d_{k,j}]$) from neighboring robots, R_k tries to refine its actual pose domain, by propagating the new distance constraints between R_k and each of its neighbours. A CSP is also built and SIVIA is used to refine the pose domain.

Experimental Results

The proposed method has been tested with data acquired on Parrot AR-Drone2 UAV, with 5 landmarks represented by AprilTag markers. The image measurement error bounds are set to ±0.5 px and the range measurement error is assumed to be within ±5 cm.
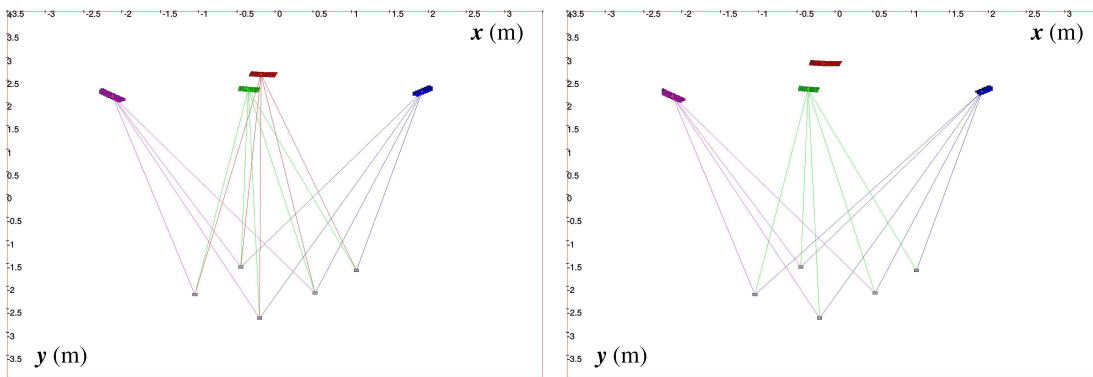
Figure 1: Pose domain for 4 robots.

The left part of Fig. 1 shows subpavings obtained when all 4 robots observe the 5 landmarks (full visibility case). The image on the right of Fig. 1 shows how cooperative localization reduces the feasible pose domain when one robot (in red) cannot clearly see the landmarks, by propagating position information of the neighbours. The average horizontal position error is less than 5 cm for each of the drones.

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A Robust Fault Detection Method Using a Zonotopic Kaucher Set-Membership Approach - Application to a Real Single-Tank Process

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Introduction

Standard system identification methods provide only an estimation of the nominal model but do not provide a reliable means for bounding the uncertainty associated with the model. Recently some methodologies that provide a model with its uncertainty have been developed but thinking always in its application to control. The term of *Robust System Identification* is used to describe the new methodologies of system identification that provide not only a nominal model but also a reliable estimate for the uncertainty associated with the model. In the *Fault Detection and Identification* (FDI) community, [1] have suggested an adaptation of classical system identification methods in order to provide the nominal model plus the uncertainty bounds for parameters that guarantee that all collected data from the system in non-faulty scenarios will be included in the model prediction interval. Those methods use zonotopes to enclose an outer approximation of the feasible parameters. However, in the setting of safety critical systems, the worst-case view needs to be complemented by regarding also

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type II errors to guarantee correct functional behavior. A method that will only verify a system if the real behavior is given by parameters within the nominal parameter set was presented at SWIM SMART 2017 and published in [3]. The main difference to [1] is that an inner approximation of the feasible parameter set is used instead of an outer approximation. Kaucher interval arithmetic is used to enclose measurement noise with known properties leading to guaranteed verification of the system behavior. A robust fault detection method was developed by combining previous work of the authors [2]. The new method uses Kaucher arithmetic to define the feasible solution set as introduced in [3]. This set is initially bounded by a zonotopic outer enclosure which is then shrunken as proposed by [1] to achieve a zonotopic inner enclosure. The shrinking is done here by interpreting the Kaucher representation of the measurement data as constraints of an optimization problem. The feasibility of a zonotope with respect to all constraints can be checked efficiently by using the Prager-Oettli theorem now applied to all vertices of the zonotope. The result of the optimization is an area maximal zonotopic inner enclosure of the united solution set given by the measurement data. The proposed approach is applied to measurement data obtained from a real single-tank process.

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Interval State estimation With Data Association

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Keywords: Interval analysis, robotics, localization

In this paper, we consider a state estimation problem in the case where we have to solve the data association problem. This problem can be formalized by the following state equations:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)\right), \mathbf{u}(t)) & \text{(evolution equation)} \\ \mathbf{g}(\mathbf{x}(t), \mathbf{y}(t)) \in \mathbb{M} & \text{(observation constraint)} \\ \mathbf{x}(0) \in \mathbb{X}_0 & \text{(initial state)} \end{cases} \quad (1)$$

where $\mathbf{x}$ is the unknown state vector, $\mathbf{y}$ an output measurement vector, $\mathbf{u}$ an input measurement vector. It is known that for all t, $\mathbf{u}(t) \in [\mathbf{u}]$ and $\mathbf{y}(t) \in [\mathbf{y}]$. The state vector $\mathbf{x}(t)$ is consistent with the interval measurement $[\mathbf{y}](t)$ if

$$\exists \mathbf{m}(t) \in \mathbb{M}, \exists \mathbf{y}(t) \in [\mathbf{y}](t), \mathbf{m}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{y}(t)).$$

The existential quantification $\exists \mathbf{m}(t) \in \mathbb{M}$ highlights the requirement of solving the so-called *data association problem* which aims at finding which point of $\mathbb{M}$ is associated with the measurement vector $\mathbf{y}$. If $\mathbb{M}$ is composed with finite number of isolated points. Our problem copes with the initial localization problem on a field of point landmarks that are indistinguishable . All measurements have the same aspect and cannot be associated directly with a particular point of the map. This problem frequently arises when acoustic sensors are used to detect underwater environmental features. In this paper, we propose

an interval-based method to solve the localization problem efficiently [1],[2].

As an application, we will consider an underwater robot starting its mission with a huge position uncertainty ans illustrated by Figure 1. For operational reasons, no external positioning system, such as acoustics beacons or USBL, are deployed. We assume that a part of the mission area has been previously mapped during a previous survey and this area is large enough to be reached by the AUV. The corresponding map $\mathbb{M}$ describing this area is modeled by a set of 280 point landmarks. Our robot performs a small mission pattern as depicted in Figure 1. It senses its environment using a forward-looking sonar oriented toward the seabed, the scope of which is represented by the blue pie. Every three seconds, it is able to measure the distance and bearing between its pose to some landmarks which range between 10 and 70 meters. The positions of the detected landmarks are depicted by green dots. The 90 red segments represent the measurements. Note that only a small number of mapped landmarks have been detected.

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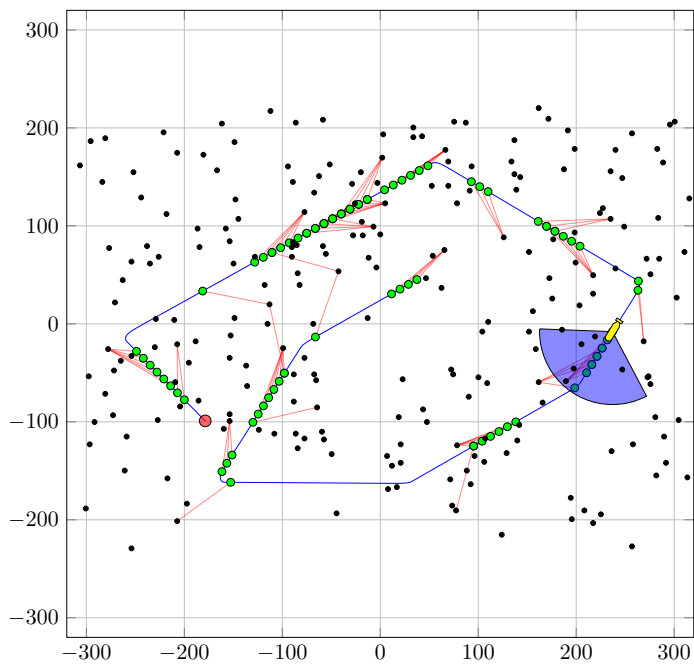[2] L. Jaulin and B. Desrochers, *Robust localisation using separators*, COPROD 2014.

Figure 1: The simulated environment for initial localization. The trajectory of the AUV is depicted by the blue lines. Its starting point is drawn by the red dot. The map is composed of 230 landmarks represented by black dots.

Observability of Nonlinear Systems Using Interval Analysis

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Keywords: Observability, Lie Derivatives, Nonlinear Systems

Introduction

An approach is presented to determine the observability of nonlinear systems on a given hyperrectangle using interval methods. In case the nonlinear system is not entirely observable on a given state space, the local observability is provided by the algorithm. In this case, the algorithm delivers all hyperrectangles which are indistinguishable. For those observability cannot be proven.

Concept and Approach

Consider a nonlinear system which can be described by the set of ordinary differential equations (ODEs)

$$\dot{x}\left(t\right) = f\left(x\left(t\right)\right), \quad x\left(0\right) = x_0 \quad \text{and} \quad y = h\left(x\left(t\right)\right)$$

with $f\colon\mathbb{R}^n \to \mathbb{R}^n$ and the output $h\colon\mathbb{R}^n \to \mathbb{R}^m$ as real-analytic functions. To determine the observability of such a nonlinear system the distinguishability of the observability function defined by the Lie derivatives

$$L_f^k h\left(x\right) = \frac{\partial}{\partial x}\left(L_f^{k-1} h\left(x\right)\right)\, f\left(x\right)$$

is investigated, with $k = 1, \ldots, \kappa$ and $L_f^0 h\left(x\right) = h\left(x\right)$. The Lie derivatives are computed automatically by using (the corresponding) power

series. Therefore, it is not necessary to solve the Lie derivatives itself. The amount of Lie derivatives κ cannot be determined in advance and depends on the nonlinear system. However, the presented iterative interval-based approach is able to adjust κ dynamically.

Due to the use of interval methods distinguishability and therefore observability cannot be proven through the observability function alone. Since the algorithm bisects the initial hyperrectangle further and further, neighboring hyperrectangles will always have at least one common point. However, this issue can be resolved by checking the local condition

$$\text{rank}\left(\frac{\partial q\left(x\right)}{\partial x}\right) = n$$

with $q\left(x\right)$ being the observability function. The Jacobian of $q\left(x\right)$ is computed with automatic differentiation used in the power series expansion. Furthermore, the rank condition is checked by computing the eigenvalues of the Jacobian. The interval eigenvalues are enclosed with a method provided by [3].

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State Estimation and Control Design for Cooperative Dynamic Systems

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Introduction

A huge number of dynamic models for heat transfer systems, resulting for example from a finite volume semi-discretization of the underlying partial differential equations, are naturally characterized by the property of cooperativity [2,3]. This property reflects the monotonicity of the state trajectories, both with respect to uncertain initial conditions and uncertain system parameters. If a worst case range enclosure of these uncertain quantities can be specified during system modeling or by means of interval-based global optimization procedures during parameter identification, the property of cooperativity can be exploited to easily compute guaranteed lower and upper bounds for each of the system states at least in an open-loop manner, where the system inputs are given a-priori as time-dependent expressions.

However, the situation becomes more complex if observer-based state estimation procedures are to be applied for such systems. In order to preserve the cooperativity of the uncertain system model, it is necessary that the choice of the observer gain matrix, which superimposes a weighted difference between the measured and estimated system outputs (both containing interval uncertainty) onto the open-loop system dynamics, does not destroy the property of cooperativity [2].

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Main Results

This contribution describes recent results for the design of cooperative state observers for uncertain linear systems $\dot{\mathbf{x}}_i(t) = \mathbf{A}_i\mathbf{x}_i(t) + \mathbf{B}_i\mathbf{u}(t)$, $\mathbf{A}_i \in \left[\underline{\mathbf{A}}_i \; ; \; \overline{\mathbf{A}}_i\right]$, $\mathbf{B}_i \in \left[\underline{\mathbf{B}}_i \; ; \; \overline{\mathbf{B}}_i\right]$ which are described by a union of individual parameter-dependent system representations $i \in \{1, \ldots, L\}$, where each of the submodels itself depends on the interval parameter vectors $\mathbf{p}_i \in \left[\underline{\mathbf{p}}_i \; ; \; \overline{\mathbf{p}}_i\right]$. Firstly, guaranteed lower and upper bounds for all states $\mathbf{x}_i(t) \in [\mathbf{v}_i(t) \; ; \; \mathbf{w}_i(t)]$ compatible with each of the L submodels and corresponding measurements $\mathbf{y}_{\mathrm{m}}(t) \in [\mathbf{y}_{\mathrm{m}}](t) = \mathbf{C}\mathbf{x}(t) + [-\Delta\mathbf{y}_{\mathrm{m}} \; ; \; \Delta\mathbf{y}_{\mathrm{m}}]$ with interval tolerances are estimated in terms of the observer outputs for which the enclosure property $[\mathbf{v}_i(t) \; ; \; \mathbf{w}_i(t)] \subseteq [\hat{\mathbf{v}}_i(t) \; ; \; \hat{\mathbf{w}}_i(t)]$ holds. Secondly, the estimates are employed to implement closed-loop controllers $\mathbf{u} = -\sum_{i=1}^{L} \left(\underline{\mathbf{K}}_i\hat{\mathbf{v}}_i + \overline{\mathbf{K}}_i\hat{\mathbf{w}}_i\right)$ as a natural extension of classical linear feedback approaches [1] stabilizing the desired operating point $\mathbf{x} = \mathbf{0}$ in a guaranteed way despite bounded uncertainty in the system model. The associated control design (aiming again at a preservation of cooperativity to easily predict the range of all reachable states) is highlighted for a prototypical heat transfer test rig available at the Chair of Mechatronics at the University of Rostock.

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Transformation of Dynamic Systems Into a Cooperative Form to Exploit Advantages in Interval-Based Controller Design

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The design of interval observers for systems with cooperative state equations has already been published, for example, in [1] and [2]. Here, the property of cooperativity is regarded as an efficient approach to handle uncertain dynamic system models as well as uncertain measurements, when these uncertainties are bounded by interval variables. If an uncertain dynamic system is cooperative, several tasks can be highly simplified, such as the computation of guaranteed state enclosures, the design of interval observers, forecasting worst-case bounds for selected system outputs in predictive control, and the identification of unknown parameters. Cooperative systems result naturally for many models in biological, chemical, and medical applications. However, there is also a large number of systems (typically from the fields of electric, magnetic, and mechanical applications) which do not show this property if the state equations are derived using first-principle techniques. To exploit the advantages of cooperativity in these cases too, we aim at transforming such system models into an equivalent cooperative form. Unfortunately, these transformations are often not straightforward, especially, if linear systems and nonlinear ones with state-dependent system matrices are subject to bounded parameter uncertainty. This matter was already discussed in [3], where a distinction between a time-invariant transformation for systems with purely

real eigenvalues as well as time-varying transformation in the case conjugate-complex eigenvalues was made. In this presentation, however, we want to consider a system with a mixture of both types of eigenvalues. For this, we make use of the work in [3], where it was shown, that typically there exists no time-invariant transformation for conjugate-complex eigenvalues. However, the time-varying transformation needs to assume that domains of conjugate-complex eigenvalues are strictly decoupled from uncertain real eigenvalues so that an intermediate transformation into the real-valued Jordan canonical form exists (forbidding break-away points of the root locus from the real axis into the complex domain). If this is the case, not only the existence of a transformation matrix is proven, but also the system can be decoupled into two systems, one with purely real and the other with conjugate-complex eigenvalues. The known procedures of [3] are applied to find two separate suitable transformation matrices and both decoupled systems can be transformed into a cooperative form individually with less conservatism than handling all eigenvalues by the time-varying transformation. Finally, the system can be re-combined into a full cooperative form. The procedure is tested on a stacker crane with interval parameters as a real-life application scenario.

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Reliable Propagation of Time Uncertainties in Dynamical Systems

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Keywords: State Estimation, Time Uncertainties, Nonlinear Systems, Tubes, Robotics, Constraints, Contractors

Contractor Programming

Our problem is to completely deal with dynamical state estimation by using a constraint programming approach. In a nutshell, the method consists in breaking state equations into a set of elementary constraints that must be satisfied by the variables of the problem. In our case, the constraints may be non-linear or differential equations and the variables are vectors (*e.g.* $\mathbf{z} \in \mathbb{R}^n$) or trajectories (*e.g.* $\mathbf{x}(\cdot) \in \mathbb{R} \to \mathbb{R}^n$). The variables are known to belong to some domains. For vectors of $\mathbb{R}^n$, we will use boxes in $\mathbb{IR}^n$. For trajectories, we will use *tubes* denoted by $[\mathbf{x}](\cdot):\mathbb{R} \to \mathbb{IR}^n$. Constraints will be applied on these domains by means of operators $\mathcal{C}$ called *contractors* [2].

Contribution

State estimation usually involves algebraic and differential constraints on trajectories such as $a(\cdot) = \sin\big(b(\cdot)\big)$ or $\dot{x}(\cdot) = v(\cdot)$. The related contractors have been the subject of some work. It remains to deal with the following elementary *evaluation* constraint denoted by $\mathcal{L}_{\text{eval}}$:

$$\mathcal{L}_{\text{eval}}\big(t, z, y(\cdot), w(\cdot)\big):\big\{z = y(t) \ , \ \dot{y}(\cdot) = w(\cdot)\big\} \tag{1}$$

with $t \in [t], z \in [z], y(\cdot) \in [y](\cdot), w(\cdot) \in [w](\cdot)$. Here, $w(\cdot)$ is the derivative of the signal to be evaluated. The problem is complex as it may involve time uncertainties related to $[t]$ that are difficult to propagate through the differential equation.

We propose the related contractor $\mathcal{C}_{\text{eval}}$, see [1], that will reliably reduce the sets of feasible solutions by contracting the bounds of the tube $[y](\cdot)$ and the intervals $[t]$ and $[z]$. Figure 1 provides an illustration of the evaluation of a trajectory in a bounded-error context.
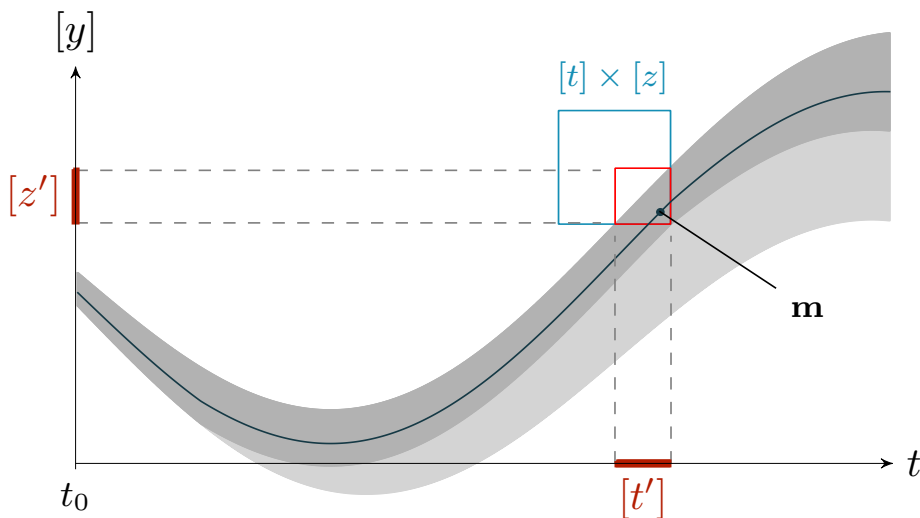

Figure 1: Evaluation on a tube $[y](\cdot)$. A given measurement $\mathbf{m} \in \mathbb{R}^2$, pictured by a black dot, is known to belong to the blue box $[t] \times [z]$. The tube is contracted by means of $\mathcal{C}_{\text{eval}}$; the contracted part is depicted in light gray. Meanwhile, the bounded observation itself is contracted to $[t'] \times [z']$ with $[t'] \subseteq [t]$ and $[z'] \subseteq [z]$. This is illustrated by the red box. The dark line is an example of a compliant trajectory.

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VERICOMP: Comparing and Recommending Verified IVP Solvers

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Keywords: Verified IVP Solvers, VERICOMP, Comparison

Research on methods with result verification [1] continues for over half a century and many open source libraries implementing the concepts are available as of now. However, engineers rarely apply them to deal with uncertainty or to verify their computations. One of the reasons is the lack of information which of the accessible tools to use for a given problem to achieve the best possible result. Sometimes the choice of the wrong approach can lead to conservative results discouraging the use of the whole branch of methods. The goal of the platform VERICOMP [2] is to make the situation better at least in one area, namely, for verified initial value problem software for ordinary differential equations (IVPS). Additionally, it offers developers of verified IVPS a possibility to compare their solvers with the established ones. VERICOMP can be of use here for facilitating such projects as ARCH-COMP 2018 (cps-vo.org/group/ARCH/FriendlyCompetition), a competition on verifying continuous and hybrid systems.

Differential equations are indispensable as a mathematical model for dynamic systems or processes in many applied areas of science. Developing methods for comparing IVP solvers based on traditional, non-verified techniques has been an important task at least since the nineteen seventies [3]. The general goal of such comparison systems is to highlight advantages of various tools. There are a lot of challenging tasks that need solving in order to achieve this goal. A standard set of

problems has to be developed along with a set of fair criteria. Moreover, means of presenting the gathered statistics need to be devised, at their best allowing for immediate grasp of the obtained knowledge. Such platforms as TEST SET (`archimede.dm.uniba.it/~testset/testsetivpsolvers/`) provide their view on the solution for floating-point based solvers. However, verified libraries for the same purpose have to be compared differently. On the one hand, the correctness of the result does not have to be assessed since the outcomes described by enclosures of the reachable states are mathematically proven to include the exact solution to the problem. On the other hand, verified algorithms have a break-down point after which no meaningful solution can be computed, something that the traditional tools do not exhibit quite as obviously.

To our knowledge, VERICOMP is the only system for automated comparison of existing verified IVPS. Formerly available under an address at the University of Duisburg-Essen, it was shut down there. Functionality pertaining to the developed problem test set will be released shortly under `vericomp.fiw.hs-wismar.de`. A substantially extended release of VERICOMP with respect to semi-automatic addition of new solvers is our long-term goal. In this contribution, we will discuss general challenges that developing such a system presents, for example, the questions of devising a fair set of criteria or a meaningful classification of available problems allowing for automatic recommendation of solvers based on the already existing results. Additionally, we will address the means of visualizing the obtained information.

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Inner and Outer Approximation of the Viability Kernel for a Bounded Uncertain System

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Abstract

In control theory, uncertainties are often added into the models to describe the behaviour of systems. For the case of non linear systems, such uncertainties produce complex dynamics that make its analysis to become more complicated and impractical. In such cases, Viability theory [1] offers an alternate view of the problem; in which by using some constrains, guaranteed integration techniques and interval analysis it is possible to obtain guaranteed regions numerically in the state space to verify if the evolution of the system remains in certain region under a set of constrains for a defined period of time, in other words to verify if a system is viable.

In this paper, we propose a method to compute an approximation of the viability kernel for the system system $\mathcal{S}$ defined by

$$
\begin{aligned}
\dot{x}(t) &= f(x(t), u(t)) + \gamma \\
u(t) &\in \mathbb{U}
\end{aligned}
\tag{1}
$$

where $x(t) \in \mathbb{R}^n$ is the system state, $\mathbb{U}$ is a compact subset of $\mathbb{R}^m$, $u \in \mathcal{U} = u{:}\mathbb{R}^+ \longmapsto \mathbb{U}$, $f{:}\mathbb{R}^n \times \mathbb{U} \longmapsto \mathbb{R}^n$ being f a continuous and locally Lipschitzian function bounded in $\mathbb{R}^n \times \mathbb{U}$, Γ is a compact subset of $\mathbb{R}^p$ and $\gamma \in \Gamma$.

The algorithm, based on interval methods, is used for obtaining the set inner ($\mathbb{V}_{in}$), which belongs to the viable set, and its complement ($\mathbb{V}_{out}$); to approximate the viability kernel inside an initial box $\mathbb{K} \subset \mathbb{R}^n$.

The implementation of this algorithm, supposes that an initial approximation of the viability kernel has been found. The initial approximation of the viable set, is defined as the boxes of the subpaving $\mathbb{V}_{in}$ that belong to the $ViabS(\mathbb{K})$ [2]. Such set can be obtained using Lyapunov theory or V-viability theory [2].

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