Karlsruhe Institute of Technology

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A Robust Fault Detection Method using a Zonotopic Kaucher Set-membership Approach - Application to a Real Single-Tank Process

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Motivation

- State of the art verification
  - Time and resource consuming
  - Based on expert knowledge

- New method needs to handle
  - Increasing system complexity
  - Strict safety requirements
  - Constraints on system dynamics
  - Guarantees

- Idea: Guaranteed verification
  of system dynamics using
  inner enclosure

Image source: www.tesla.com

Outline

Main Concept

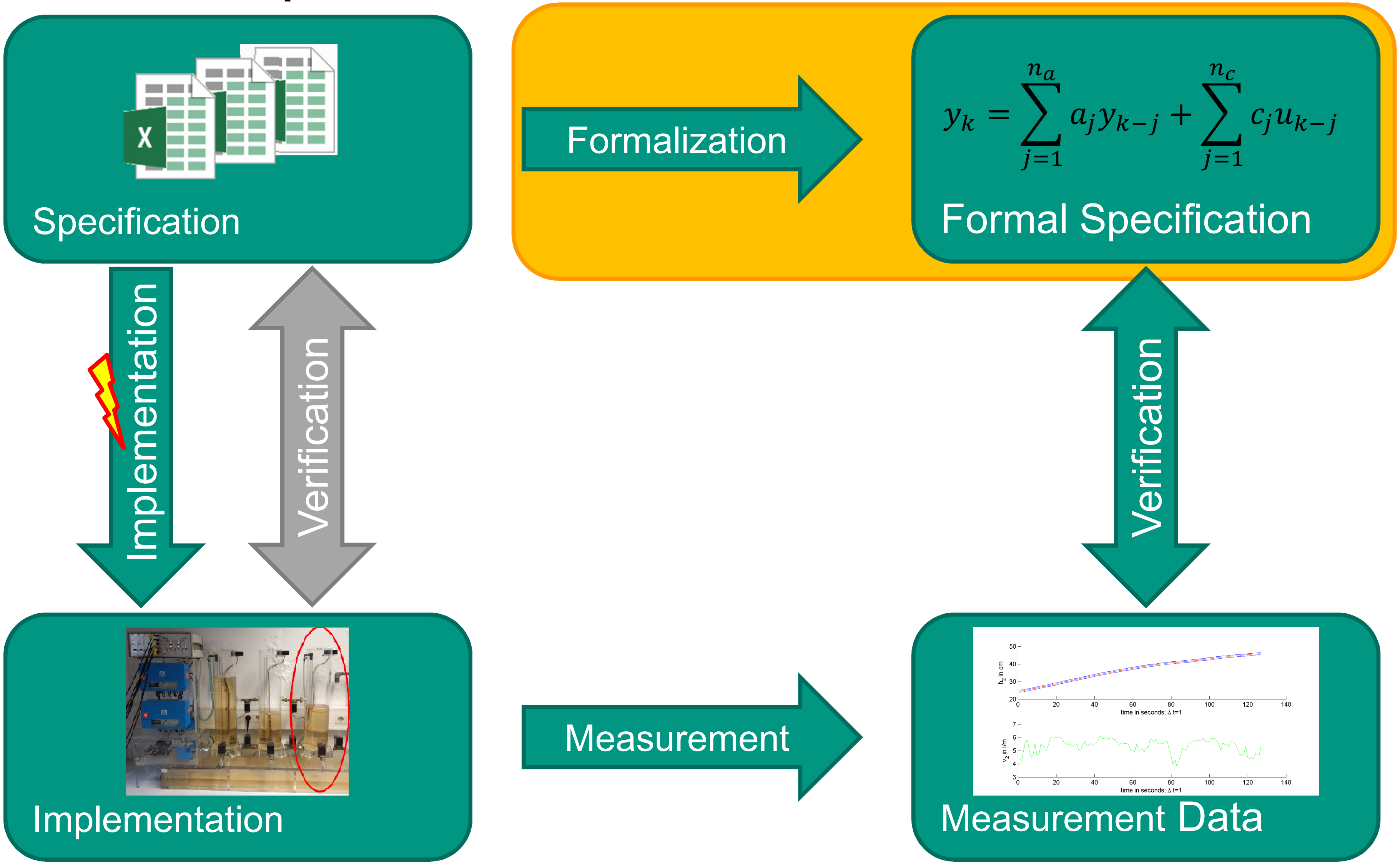
- Problem setup
- Formalization of the specification
- Measurement assumption
- Guaranteed verification
- Zonotopic enclosure

Examples

- Single Tank Process

Summary

Main Concept
Problem setup



Main Concept
Formalization of the specification

- System given in ARX form

$$y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{j=1}^{n_c} c_j u_{k-j}$$

  - Input order $\mathrm{n_a}$, output order $\mathrm{n_c}$
  - Output parameter a_j, input parameter c_j

Main Concept
Formalization of the specification

- An **interval specification** S_i^* consists of system orders $n_a^*, n_c^* \in \mathbb{R}$ and interval-type model parameters $[a_1^*] \cdots [a_{n_a^*}^*]$ and $[c_1^*] \cdots [c_{n_c^*}^*] \in I\mathbb{R}$

  - Interval-type parameters allow to express
    - Imprecise or missing knowledge
      - Modelling errors (linear model for nonlinear system)
    - Tolerances

  - given by the user
  - depicts the **combined** behavior of plant and controller

Main Concept
Problem setup

Specification

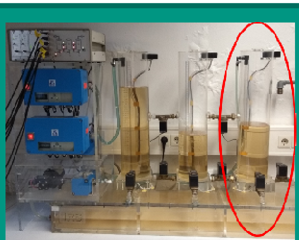
Formalisation

$$y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{j=1}^{n_c} c_j u_{k-j}$$

Formal Specification

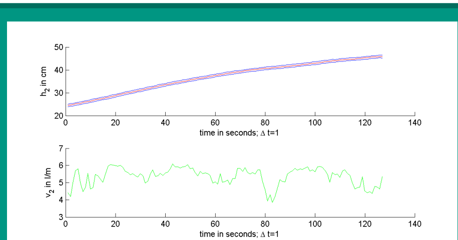
Implementation

Verification



Implementation

Measurement



Measurement Data

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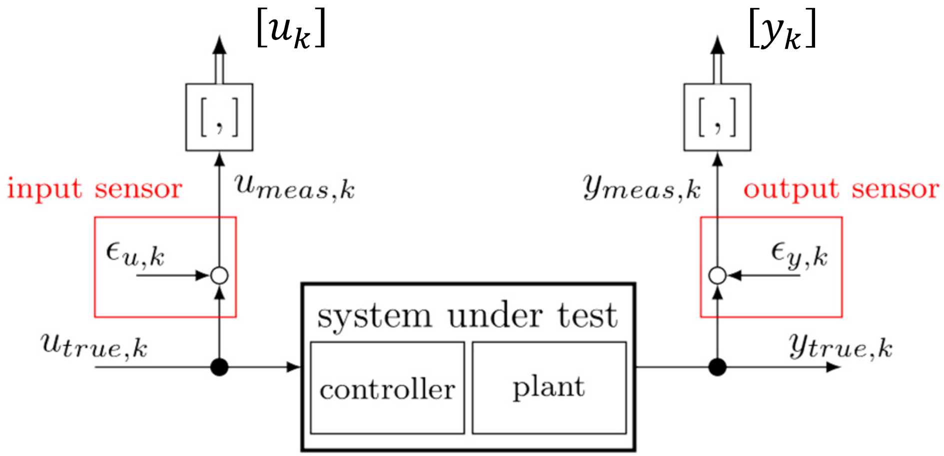
Main Concept
Measurement assumption

- Measured input and output data available for verification
- Measurement corrupted by noise



- Interval enclosure of measurement noise available
- Known sensor precision δ_{mx} as absolute value, with $x \in \{u, y\}$:

$$|\epsilon_{x,k}| \leq |\delta_{mx}|$$

$$[x_k] = [x_{meas,k} - \delta_{mx} \quad x_{meas,k} + \delta_{mx}]$$

Main concept
Measurement assumption

- Block description

$$
\begin{aligned}
&[y_{k-1}]a_1 + \cdots + [y_{k-n_a}]a_{n_a} + [u_{k-1}]c_1 + \cdots + [u_{k-n_c}]c_{n_c} = [y_k] \\
&[y_{k+1-1}]a_1 + \cdots + [y_{k+1-n_a}]a_{n_a} + [u_{k+1-1}]c_1 + \cdots + [u_{k+1-n_c}]c_{n_c} = [y_{k+1}] \\
&\quad\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad \vdots \\
&\underbrace{[y_{T-1}]a_1 + \cdots + [y_{T-n_a}]a_{n_a} + [u_{T-1}]c_1 + \cdots + [u_{T-n_c}]c_{n_c}}_{[\boldsymbol{R}]\boldsymbol{x}} = \underbrace{[y_T]}_{[\boldsymbol{d}]}
\end{aligned}
$$

- Regressor matrix $[\boldsymbol{R}]$, parameter vector $\boldsymbol{x}$, measurement vector $[\boldsymbol{d}]$

Main Concept
Problem setup

Specification

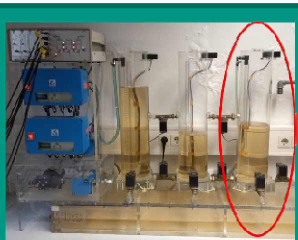
→ Formalisation →

$$y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{j=1}^{n_c} c_j u_{k-j}$$

Formal Specification

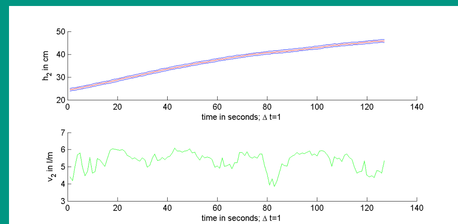
↓ Implementation

↕ Verification



Implementation

→ Measurement →



Measurement Data

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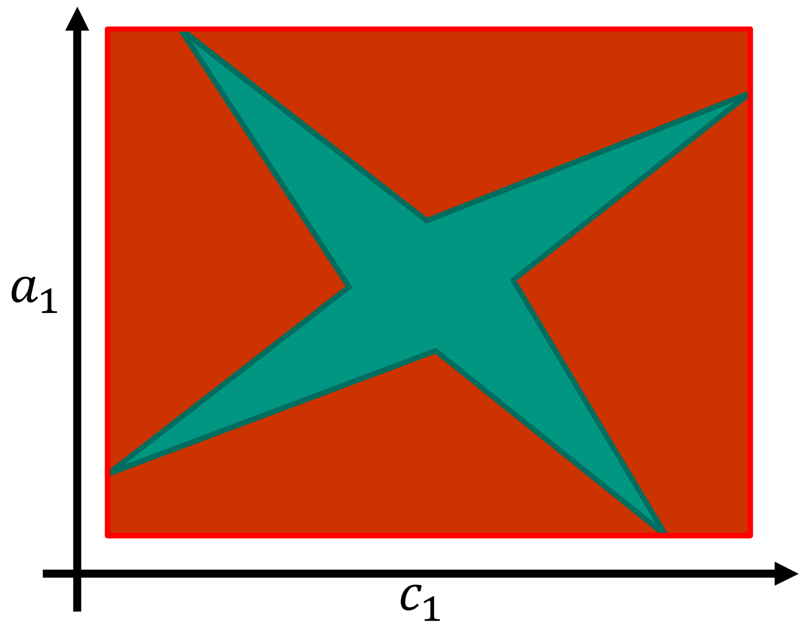
Main Concept
Guaranteed verification

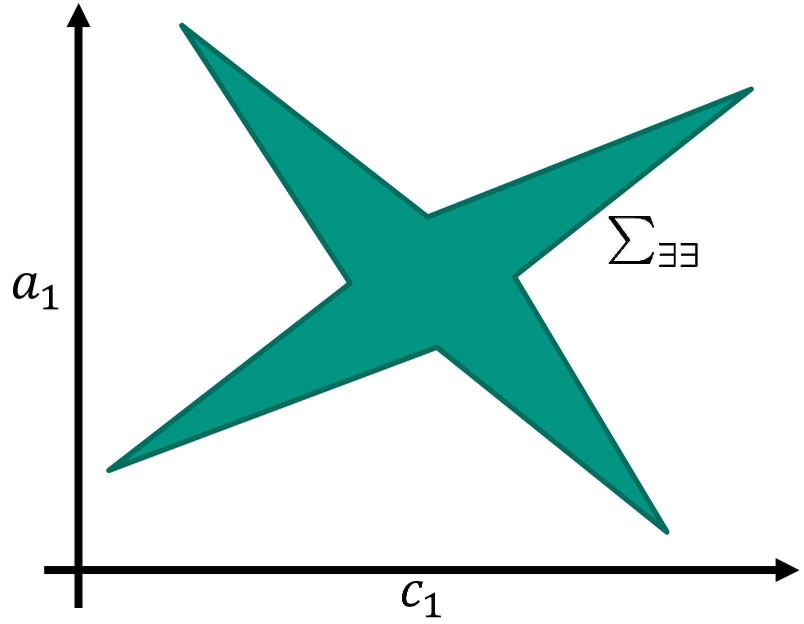
- Definition: “Consistency”
  - „The measurement data can be explained by the specification“



- Classical interval solution of $[\boldsymbol{R}]\boldsymbol{x} = [\boldsymbol{d}]$ given by interval set $[\boldsymbol{x}]$
  - True solution
  - Spurious solution

Main concept
Guaranteed verification

- Guaranteed verification
  - No false negatives
  - Only nonspurious behavior
  - Inner enclosure necessary



Source: Shary, *Algebraic approach*[2]

- United solution set $\Sigma_{\exists\exists}$
  - $\Sigma_{\exists\exists}([\boldsymbol{R}],[\boldsymbol{d}]) \coloneqq \{\boldsymbol{x} \in \mathbb{R} | (\exists \boldsymbol{R} \in [\boldsymbol{R}]), (\exists \boldsymbol{d} \in [\boldsymbol{d}]), (\boldsymbol{Rx} = \boldsymbol{d})\}$
  - Given in Kaucher interval arithmetic

[2] Shary: Algebraic approach to the interval linear static identification, tolerance, and control problems, or one more application of kaucher arithmetic. *Reliable Computing*, 3-33, 1996.

Main Concept
Verification of direct specification

- Check solution candidate $\boldsymbol{x} \in S_i^*$ using the theorem of Prager-Oettli

$$|\boldsymbol{R}_c\boldsymbol{x} - \boldsymbol{d}_c| \leq \boldsymbol{R}_\Delta|\boldsymbol{x}| + \boldsymbol{d}_\Delta$$

with

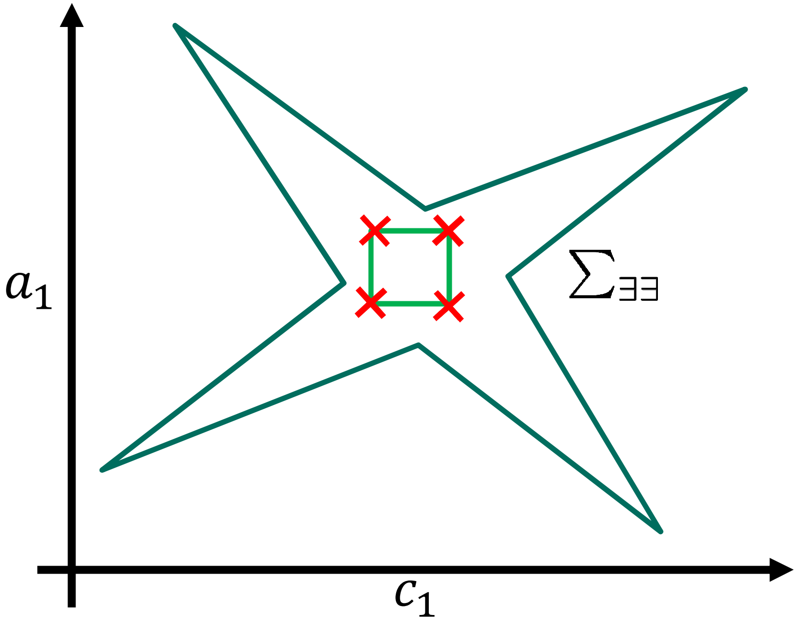
$$\boldsymbol{R}_c \in \mathbb{R}^{(l\times m)}: a_c^{(i,j)} = \frac{1}{2}\left(\overline{a}^{(i,j)} + \underline{a}^{(i,j)}\right), \text{ center matrix}$$

$$\mathbf{R}_\Delta \in \mathbb{R}^{(l\times m)}: a_\Delta^{(i,j)} = \frac{1}{2}\left(\overline{a}^{(i,j)} - \underline{a}^{(i,j)}\right), \text{ radius matrix}$$

- Prager-Oettli theorem fulfilled: $\boldsymbol{x} \in S_i^*$ is part of the united solution set $\sum_{\exists\exists}$
- Results can be obtained by the evaluation of a single criterion for each solution candidate $\boldsymbol{x}$

Main Concept
Verification of interval specification

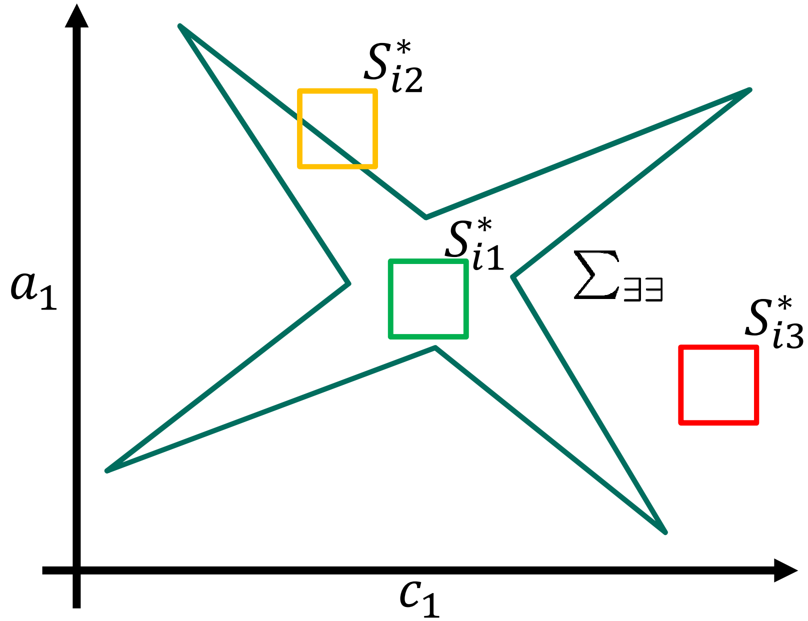
- Remaining problem: how to determine the solution candidates to check?



- Check vertexes of specification rectangle

Main Concept
Verification of interval specification

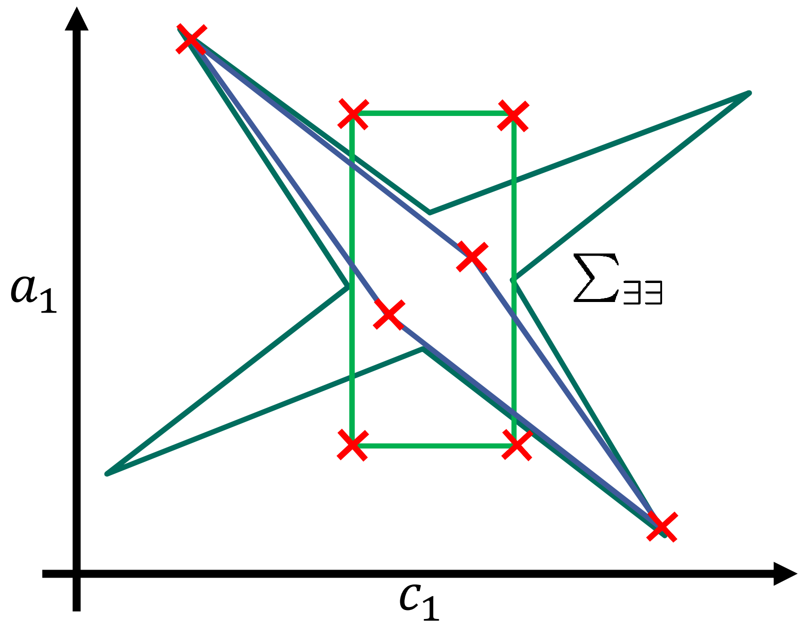
- A parameter vector $\boldsymbol{x} \in S_i^*$ can explain the measurement if there exists at least one $\exists \boldsymbol{R} \in [\boldsymbol{R}]$ and at least one $\exists \boldsymbol{d} \in [\boldsymbol{d}]$ that explains the observation



- **Full consistency** All parameter vectors $\forall \boldsymbol{x} \in S_i^*$ can explain the measurement
- **Basic consistency** At least one parameter vector $\exists \boldsymbol{x} \in S_i^*$ can explain the measurement
- **Inconsistency** No parameter vector $\exists \boldsymbol{x} \in S_i^*$ can explain the measurement

Main Concept
Verification of interval specification

- Remaining problem: how to determine the solution candidates to check?



- Initial solution: Use vertexes of rectangles[1] in optimization based approach
- Now: Use vertexes of zonotopes[2]

[1] Stefan Schwab, Oliver Stark, and Soeren Hohmann. Verified diagnosis of safety critical dynamic systems based on Kaucher interval arithmetic. Proceedings of the 20th IFAC World Congress, Toulouse, 2017.

[2] J. Blesa, V. Puig, and J. Saludes. Identification for passive robust fault detection using zonotope-based set-membership approaches. Int. Jour. of Adaptive Control & Signal Processing, 788–812, 2011.

Zonotopic Approach

- Zonotope definition

$$\sum_Z = P_0 \oplus \alpha \boldsymbol{H_0} \boldsymbol{K}^V = \{P_0 + \alpha \boldsymbol{H_0} \boldsymbol{z} : \boldsymbol{z} \in \boldsymbol{K}^V\}$$

  - $\sum_Z$ is exhaustively defined by the set of $v = 1,2,\ldots,V$ vertices
  - $P_0 \in R^{(n\times 1)}$ is the center of the zonotope
  - $H_0 \in R^{(n\times V)}$ is the radius matrix
  - K^V is a unitary box composed of V unitary interval vectors $K = [-1,1]$

Main Concept
Optimization based solution

- Objective function

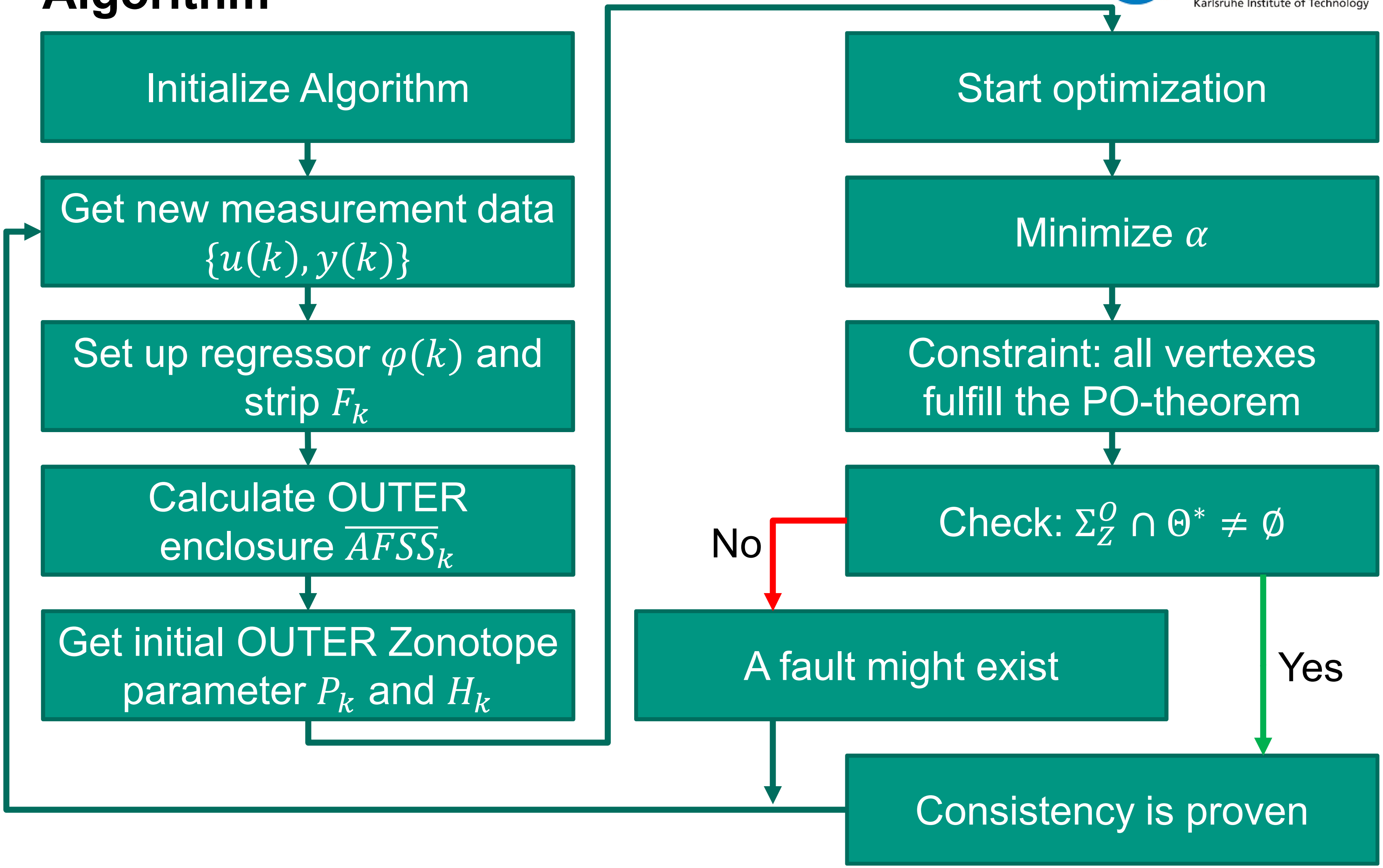
$$J(\Sigma_Z) := -\alpha$$

  - Find the most possible parameter combinations
  - Solution is not unique

- Constraints
  - All vertexes $\boldsymbol{v} = 1,2,\ldots,V$ of the zonotope are part of the united solution

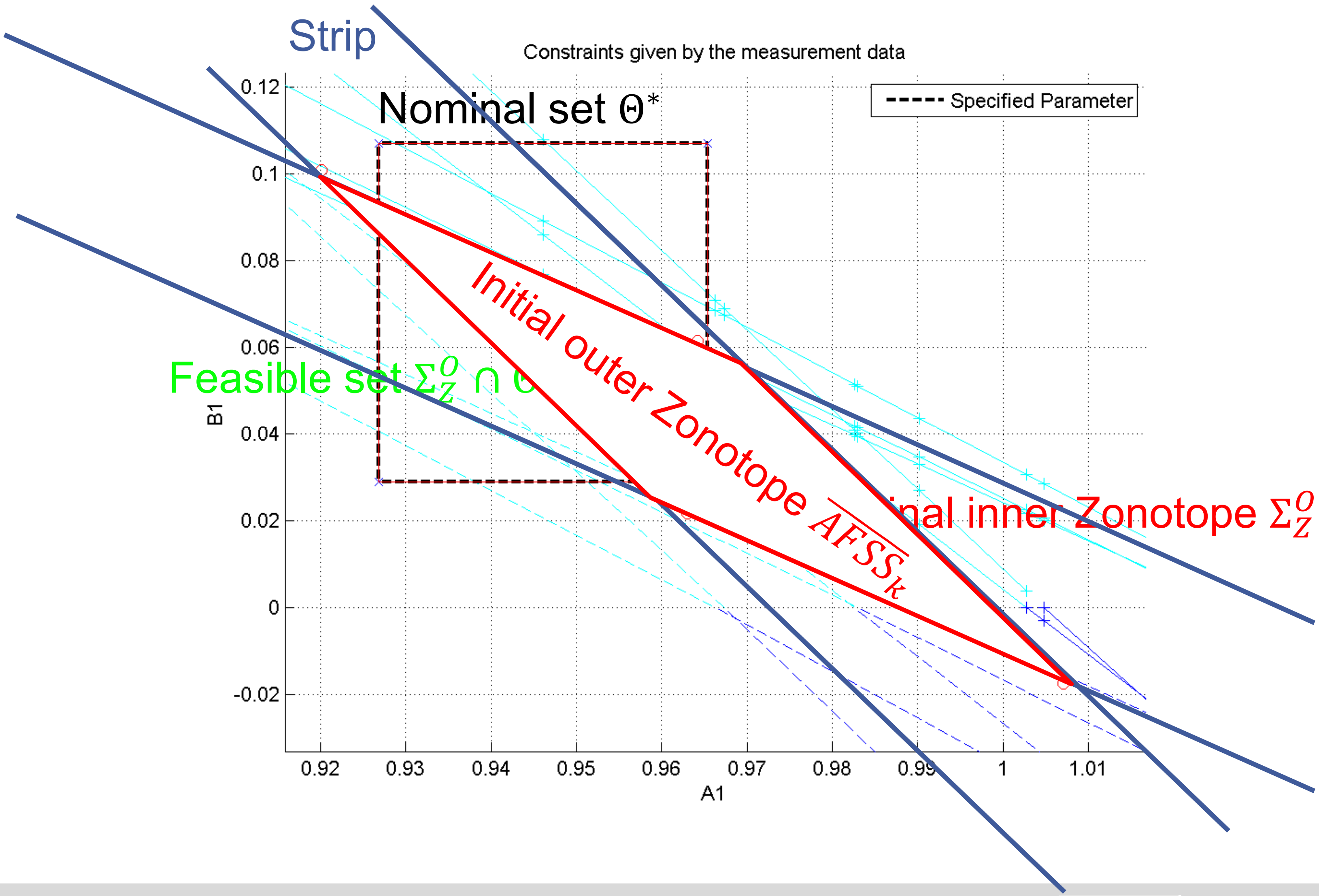
$$|\boldsymbol{R}_c\boldsymbol{v} - \boldsymbol{d}_c| \leq \boldsymbol{R}_\Delta|\boldsymbol{v}| + \boldsymbol{d}_\Delta$$

Algorithm



Initialize Algorithm
Get new measurement data $\{u(k), y(k)\}$
Set up regressor $\varphi(k)$ and strip F_k
Calculate OUTER enclosure $\overline{AFSS}_k$
Get initial OUTER Zonotope parameter P_k and H_k
Start optimization
Minimize α
Constraint: all vertexes fulfill the PO-theorem
Check: $\Sigma_Z^O \cap \Theta^* \neq \emptyset$
No
Yes
A fault might exist
Consistency is proven

Zonotopic Algorithm



Outline

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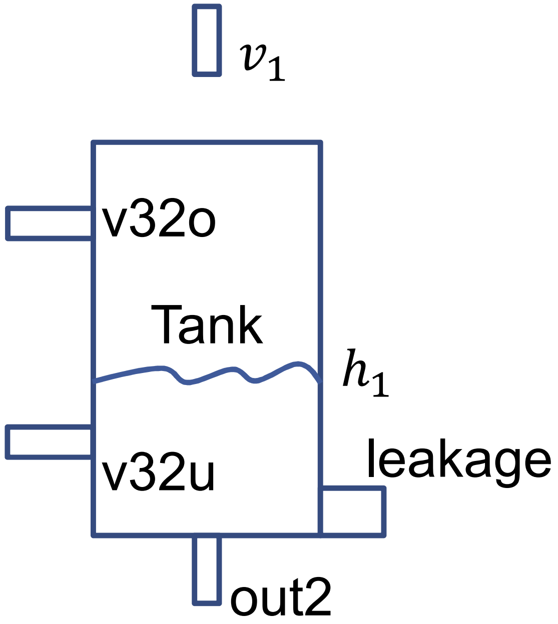
- Problem setup
- Formalization of the specification
- Measurement assumption
- Guaranteed verification
- Zonotopic enclosure

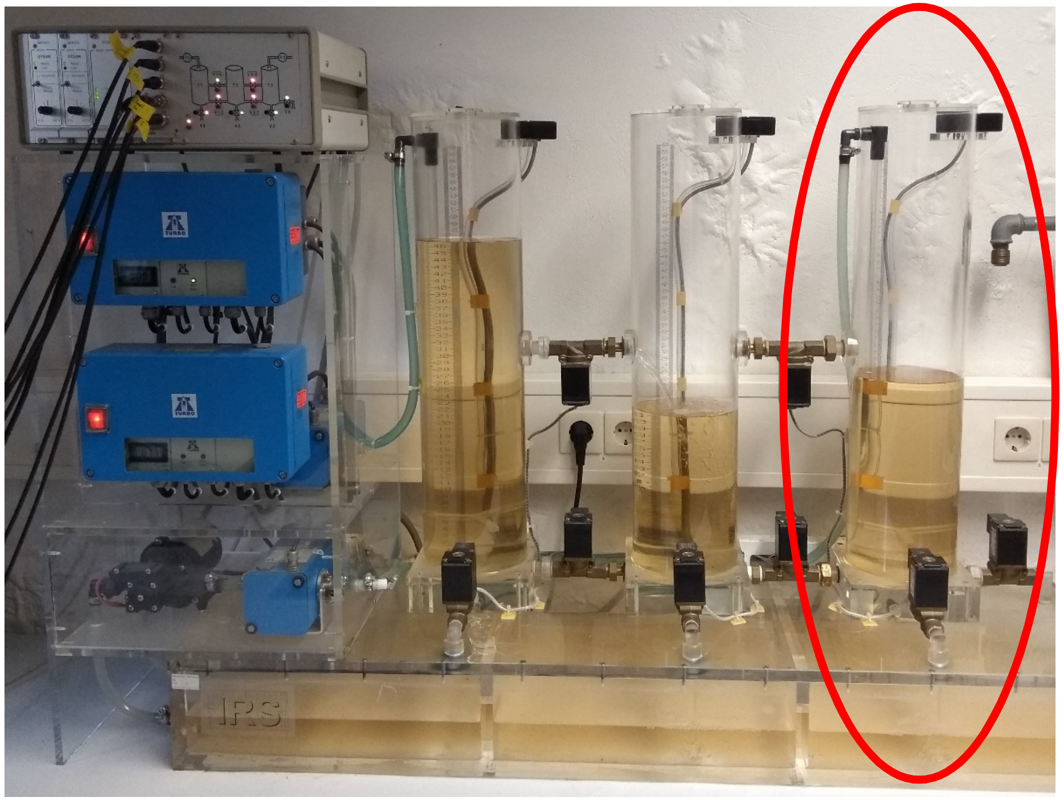
Examples

- Single Tank Process

Summary

Example: Single Tank Process





- Height h_1 measured
- Inflow v_1 measured

- Tank diameter $154cm^2$
- Pipe diameter $0,5cm^2$
- Leakage diameter $0,8cm^2$
- Max. pump flow $6l/m$

Example: Single Tank Process

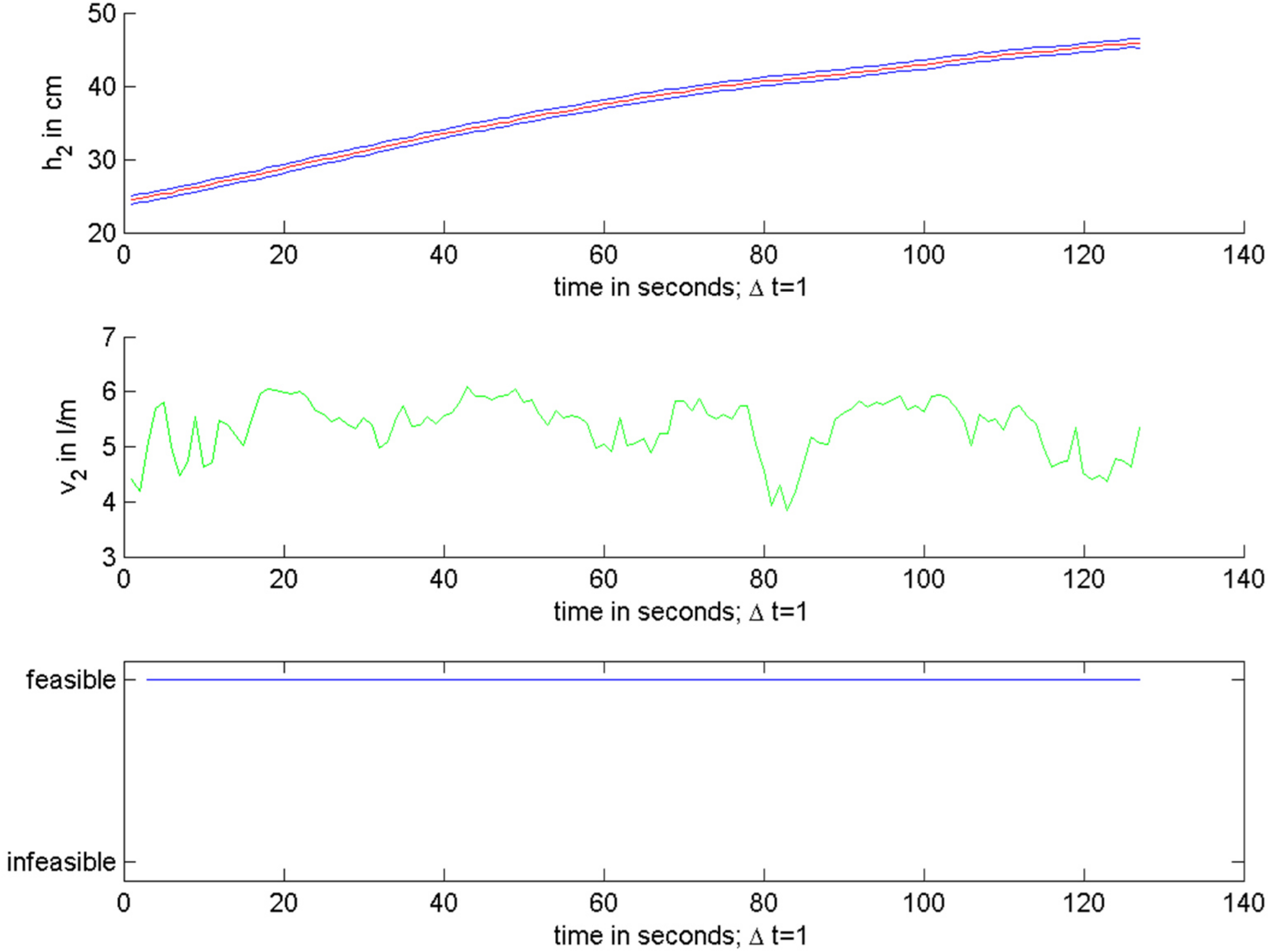
- Dynamic of h_1:
$$\frac{\mathrm{d}h_1}{\mathrm{d}t} = \underbrace{-\frac{a_1}{A_1}\sqrt{2gh_1}}_{\text{outflow}} + \underbrace{\frac{\gamma_1 k_1}{A_1} v_1}_{\text{inflow by pump 1}}$$

- Time discretization (using $\Delta t = 1$) and transformation to regressor form $y(k) = \theta(k)\varphi(k)$:

$$y(k) = h_1(k) - \frac{\gamma_1 k_1}{A_1} v(k-1)$$
$$\varphi(\mathrm{k}) = [h_1(k-1)]$$
$$\theta(k) = [A1]$$
with
$$A1(k) = 1 - \frac{a_1}{A_1}\sqrt{\frac{2g}{h_1(k-1)}}$$

for $h_1 \in [25 \quad 45]$ is $A1 \in [0.9712 \quad 0.9786]$

Failure free system



$\delta_{my} = 0.4cm$

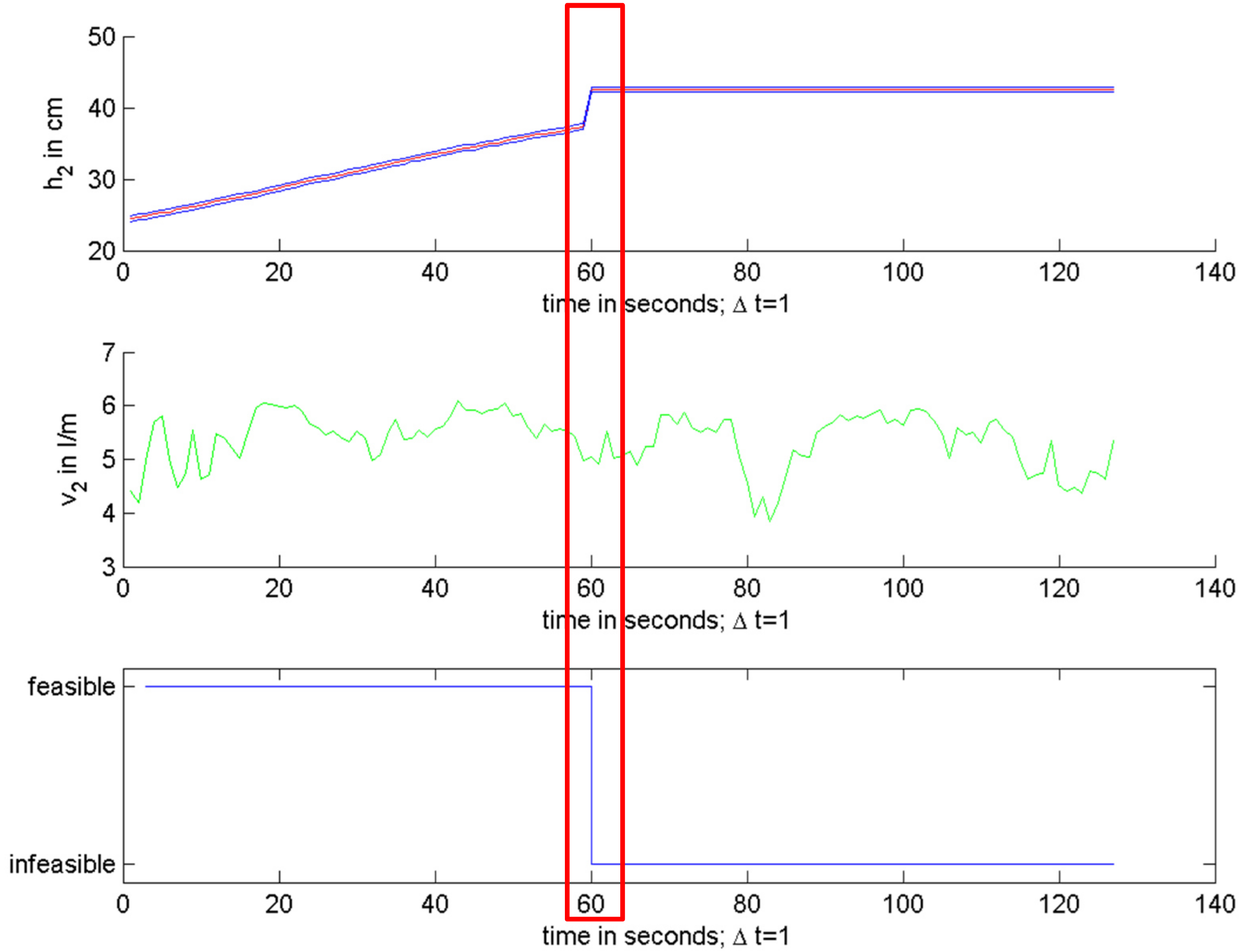
Freeze Failure - Overview

- Failure model

$$y_{err,k} = y_{k_f} + f_f, \forall k \in [k_f, \dots, T]$$

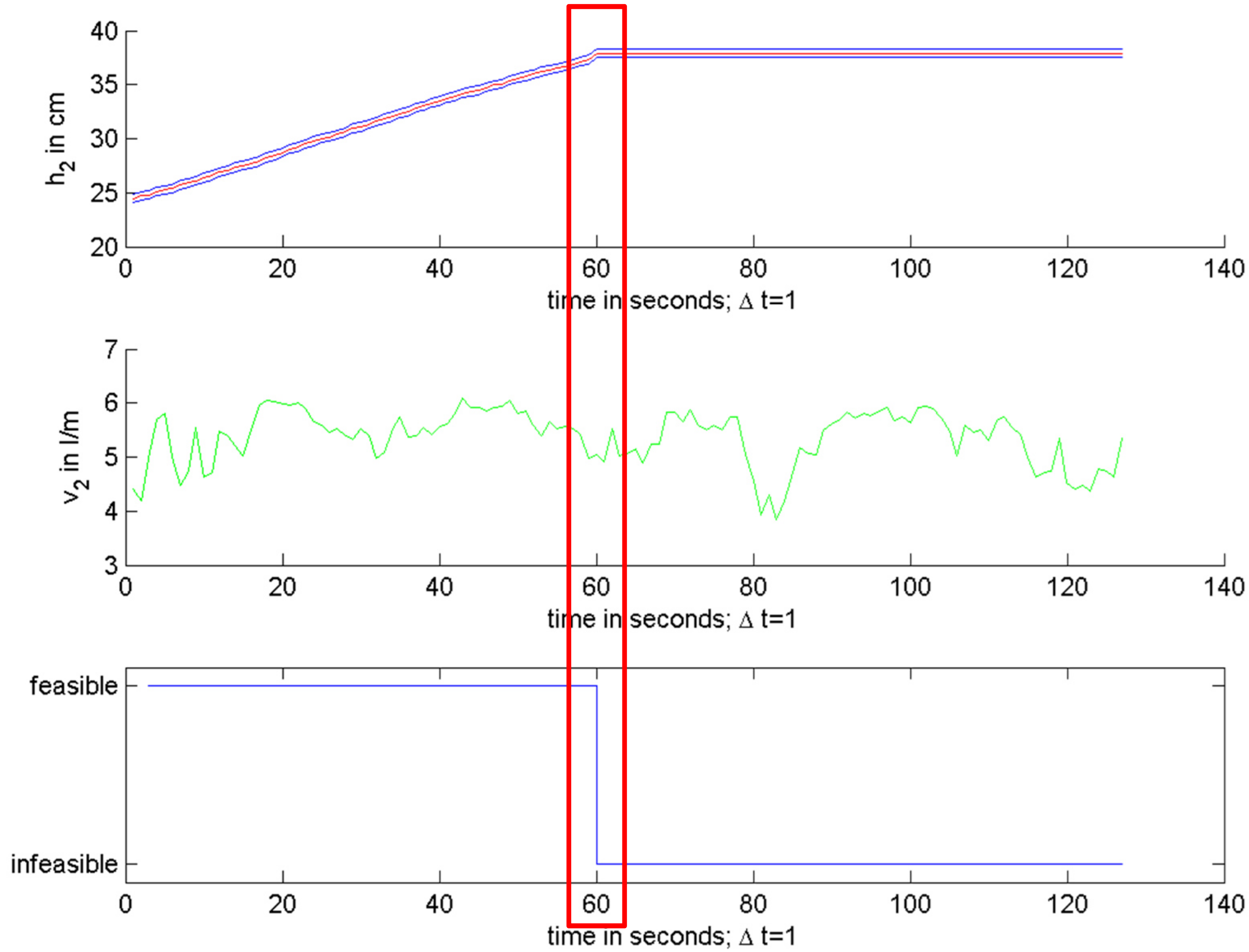
Failure f_f	Failure Time k_f	Result
+5	60	Detected
+2	60	Detected
+1	60	Detected
+0.5	60	Detected
+0.35	60	Detected
+0.2	60	Not detected

Freeze Failure



$$\delta_{my} = 0.4cm$$
$$f_f = 5cm$$

Freeze Failure



$$\delta_{my} = 0.4cm$$
$$f_f = 0.35cm$$

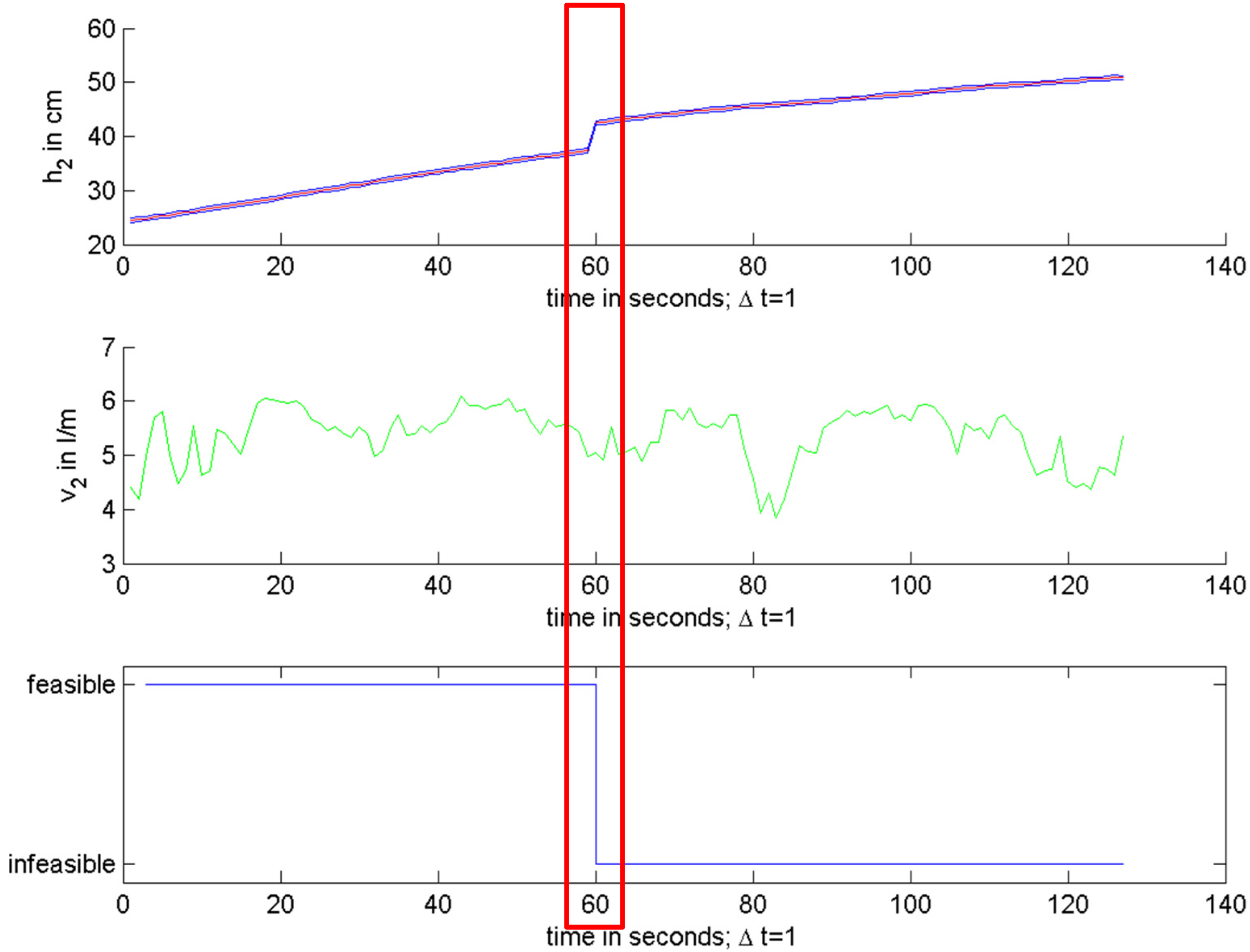
Offset Failure - Overview

- Failure model

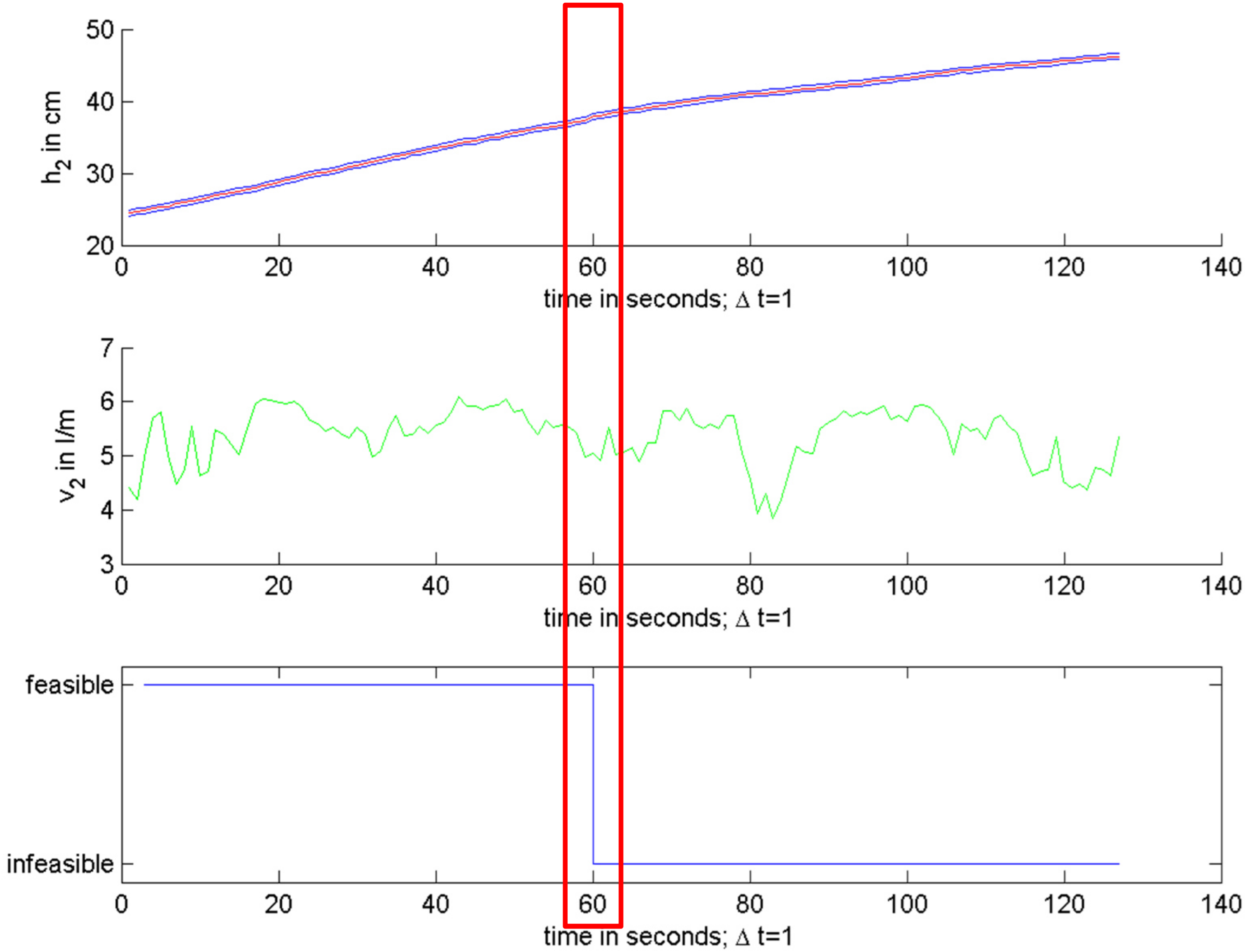
$$y_{err,k} = y_k + f_o, \forall k \in [k_f, \ldots, T]$$

Failure f_o	Failure Time k_f	Result
+5	60	Detected
+2	60	Detected
+1	60	Detected
+0.5	60	Detected
+0.35	60	Detected
+0.2	60	Not detected

Offset failure



$$\delta_{my} = 0.4cm$$
$$f_o = 5cm$$



$$\delta_{my} = 0.4cm$$

$$f_o = 0.35cm$$

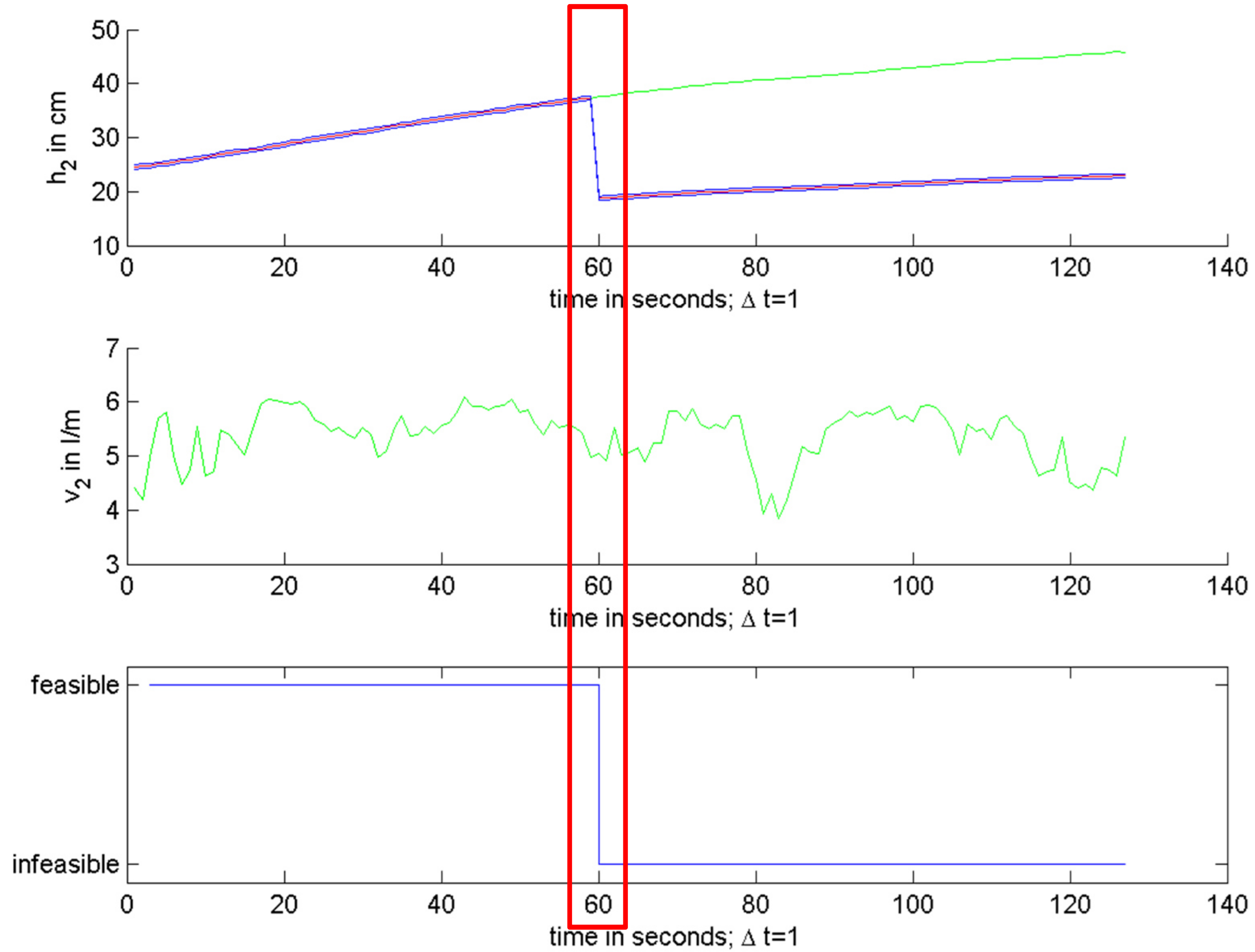
Scaling Failure - Overview

- Failure model

$$y_{err,k} = y_k \cdot f_s, \forall k \in [k_f, \dots, T]$$

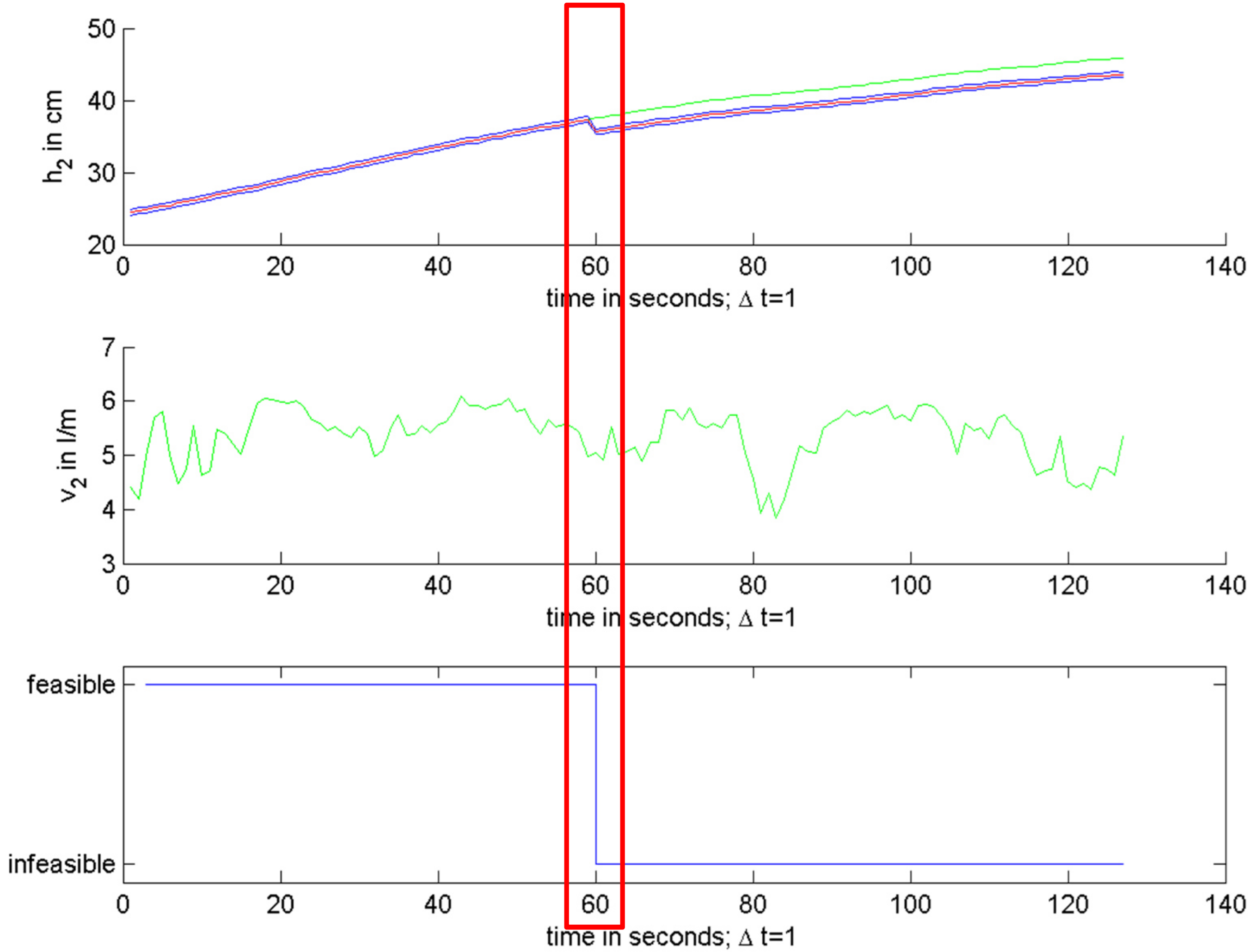
Failure f_s	Failure Time k_f	Result
0.5	60	Detected
0.75	60	Detected
0.9	60	Detected
0.95	60	Detected
0.97	60	Not detected
1.01	60	Detected
1.03	60	Detected
1.05	60	Detected
1.1	60	Detected

Offset failure



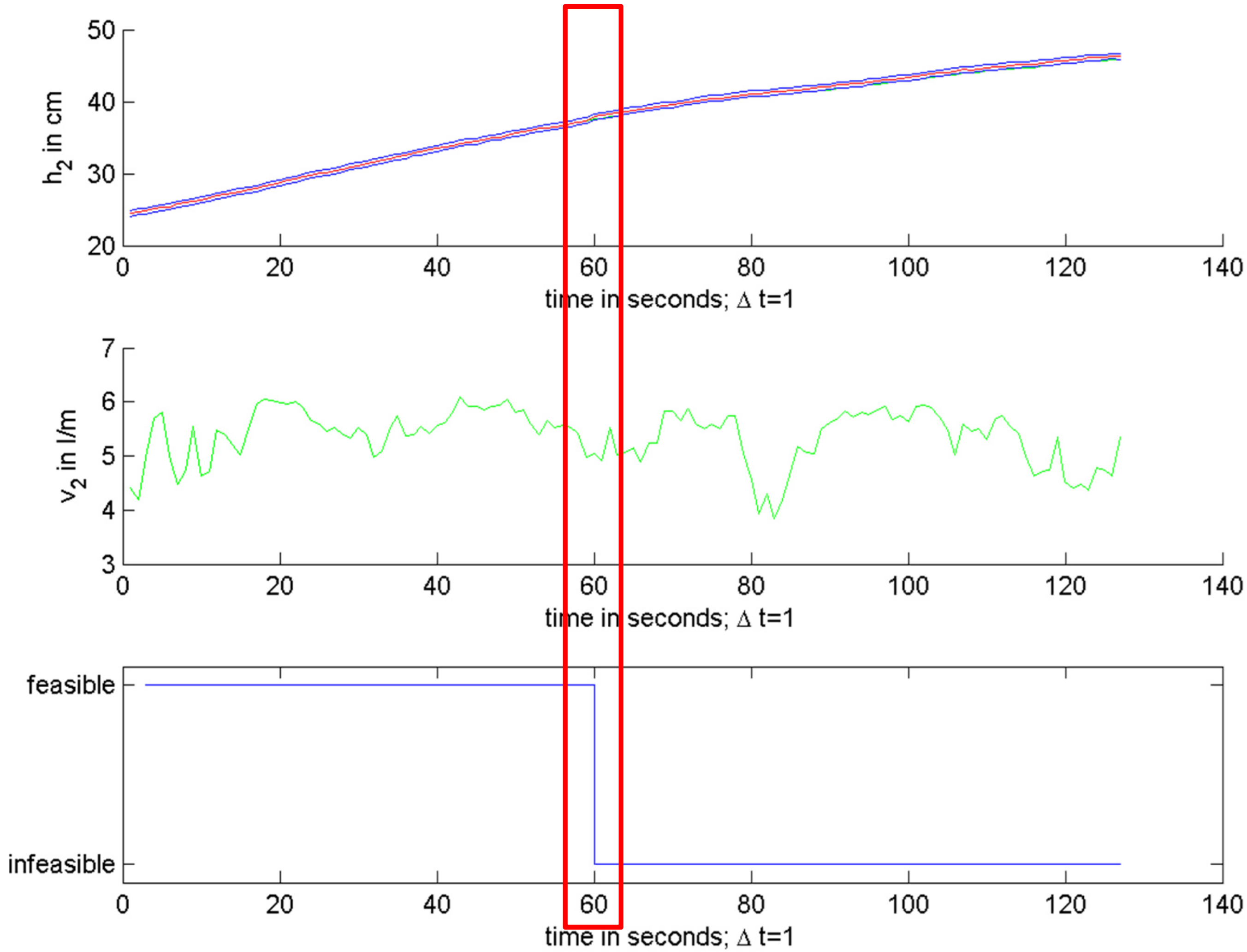
$$\delta_{my} = 0.4cm$$
$$f_s = 0.5$$

Offset failure



$$\delta_{my} = 0.4cm$$
$$f_s = 0.95$$

Offset failure



$$\delta_{my} = 0.4cm$$
$$f_s = 1.01$$

Summary

- Formalized specification defined
  - Interval specification S_i^*
  - Noise modelling
- Guaranteed verification based on united solution set
- Optimization based verification
  - Pointwise verification of vertexes
  - Zonotopic enclosure
- Application to a single tank process
  - Very small failures can be detected

Thank you for your kind attention

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