Universität Rostock Traditio et Innovatio

Chair of Mechatronics

State Estimation and Control Design for Cooperative Dynamic Systems

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Andreas Rauh, Julia Kersten, Harald Aschemann

Chair of Mechatronics
University of Rostock, Germany

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Cooperative and Positive Dynamic Systems

Simplified computation of state enclosures for cooperative systems

- Sufficient condition for cooperativity of the dynamic system

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)\right) \ , \quad \mathbf{x} \in \mathbb{R}^{n_\mathrm{x}}$$

$$J_{i,j} \geq 0 \ , \quad i,j \in \{1,\ldots,n_\mathrm{x}\} \ , \quad i \neq j \quad \text{with} \quad \mathbf{J} = \frac{\partial \mathbf{f}\left(\mathbf{x}\right)}{\partial \mathbf{x}}$$

- For initial conditions in the positive orthant

$$\mathbb{R}_+^{n_\mathrm{x}} = \{\mathbf{x} \in \mathbb{R}^{n_\mathrm{x}} \ | \ x_i \geq 0 \quad \forall i \in \{1,\ldots,n_\mathrm{x}\}\} \ ,$$

positivity of all state trajectories is ensured if

$$\dot{x}_i(t) = f_i\left(x_1(t),\ldots,x_{i-1}(t),0,x_{i+1}(t),\ldots x_{n_\mathrm{x}}(t)\right) \geq 0$$

holds for all components $i \in \{1,\ldots,n_\mathrm{x}\}$

Special Case: Linear Cooperative System with Uncertain Initial States

Interval-based representation of uncertain initial conditions

$$\mathbf{x}(0) \in \; [\mathbf{x}_0] = [\mathbf{x}](0) = \begin{bmatrix} [\underline{x}_1(0) \; ; \; \overline{x}_1(0)] \\ \vdots \\ [\underline{x}_{n_x}(0) \; ; \; \overline{x}_{n_x}(0)] \end{bmatrix} \quad \text{with} \quad \underline{\mathbf{x}}_0 \in \mathbb{R}_+^{n_x}$$

Decoupled bounding systems, $\mathbf{A}$: Metzler matrix

$$\mathbf{A} \cdot \mathbf{v}(t) + \mathbf{B} \cdot \mathbf{u}(t) = \dot{\mathbf{v}}(t) \leq \dot{\mathbf{x}}(t) \leq \dot{\mathbf{w}}(t) = \mathbf{A} \cdot \mathbf{w}(t) + \mathbf{B} \cdot \mathbf{u}(t)$$

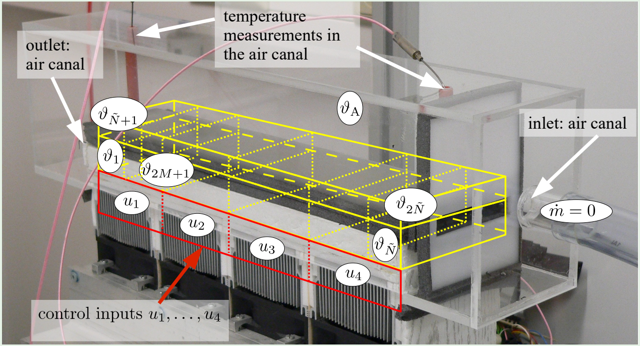
with the element-wise non-negative inputs $\mathbf{B} \cdot \mathbf{u}(t)$

Guaranteed state enclosures $\mathbf{x}(t) \in [\mathbf{v}(t) \; ; \; \mathbf{w}(t)]$

$$\mathbf{v}(0) = \underline{\mathbf{x}}_0 \quad \text{and} \quad \mathbf{w}(0) = \overline{\mathbf{x}}_0$$

Example: Spatially Distributed Heating System

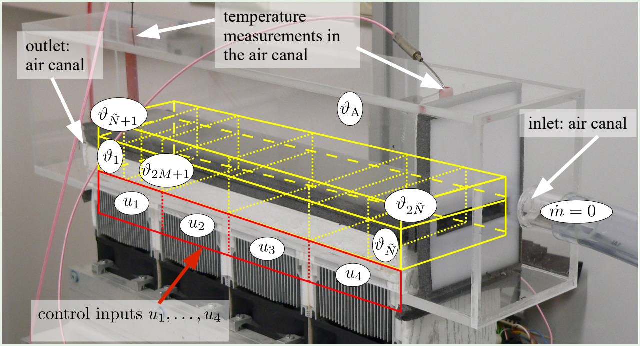
Experimental setup



- Identification of parameters for heat convection, heat conduction, and thermal air canal properties, cf. Mathmod 2018, Vienna, Austria
- Peltier elements as control inputs

Example: Spatially Distributed Heating System

Early lumping: Finite volume semi-discretization



- Differential equation for the rod temperature

$$\dot{\vartheta}_i(t) = \frac{1}{c_i \cdot m_i} \cdot \left(\dot{Q}^{\lambda,i}_{i-1}(t) + \dot{Q}^{\lambda,i}_{i+1}(t) + \dot{Q}^{\alpha,i}_{\mathrm{B}}(t) + \dot{Q}^{\alpha,i}_{\tilde{N}+i}(t) + \tilde{u}_i(t) \right)$$

Example: Spatially Distributed Heating System

Early lumping: Finite volume semi-discretization

- Differential equation for the rod temperature

$$\dot{\vartheta}_i(t) = \frac{1}{c_i \cdot m_i} \cdot \left(\dot{Q}_{i-1}^{\lambda,i}(t) + \dot{Q}_{i+1}^{\lambda,i}(t) + \dot{Q}_{\mathrm{B}}^{\alpha,i}(t) + \dot{Q}_{\tilde{N}+i}^{\alpha,i}(t) + \tilde{u}_i(t) \right)$$

with $\tilde{u}_i(t) = \frac{1}{2M+1} u_\xi(t)$, $\xi = \left\lceil \frac{i}{2M+1} \right\rceil$, $i \in \{1, \ldots, \tilde{N}\}$

- Heat conduction between neighboring elements

$$\dot{Q}_{i-1}^{\lambda,i}(t) = \lambda_{\mathrm{R}} \cdot \frac{A_{\mathrm{c}}}{l_{\mathrm{s}}} \cdot (\vartheta_{i-1}(t) - \vartheta_i(t))$$

- Heat convection between rod and air canal

$$\dot{Q}_{\tilde{N}+i}^{\alpha,i}(t) = \alpha \cdot A_{\mathrm{s}} \cdot \left(\vartheta_{\tilde{N}+i}(t) - \vartheta_i(t) \right) = -\dot{Q}_i^{\alpha,\tilde{N}+i}(t)$$

Definition of Point-Valued Bounding Systems

Decoupled bounding systems, $\mathbf{A}\left(\underline{\mathbf{p}}\right) \in \left[\mathbf{A}\left(\mathbf{p}\right) \; ; \; \mathbf{A}\left(\overline{\mathbf{p}}\right)\right]$: uncertain Metzler matrix

$$\mathbf{A}\left(\underline{\mathbf{p}}\right) \cdot \mathbf{v}(t) + \mathbf{B} \cdot \mathbf{u}(t) = \dot{\mathbf{v}}(t) \leq \dot{\mathbf{x}}(t) \leq \dot{\mathbf{w}}(t) = \mathbf{A}\left(\overline{\mathbf{p}}\right) \cdot \mathbf{w}(t) + \mathbf{B} \cdot \mathbf{u}(t)$$

with the parameter intervals $\alpha \in [\underline{\alpha} \; ; \; \overline{\alpha}]$, $\alpha_{\mathrm{B}} \in [\underline{\alpha}_{\mathrm{B}} \; ; \; \overline{\alpha}_{\mathrm{B}}]$, $\alpha_{\mathrm{T}} \in [\underline{\alpha}_{\mathrm{T}} \; ; \; \overline{\alpha}_{\mathrm{T}}]$, $\Delta\alpha \in \left[\underline{\Delta\alpha} \; ; \; \overline{\Delta\alpha}\right]$, $\Delta m_{\mathrm{a}} \in \left[\underline{\Delta m}_{\mathrm{a}} \; ; \; \overline{\Delta m}_{\mathrm{a}}\right]$, and $\lambda_{\mathrm{R}} \in \left[\underline{\lambda}_{\mathrm{R}} \; ; \; \overline{\lambda}_{\mathrm{R}}\right]$

Block-wise definition of the system matrix

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} \mathbf{A}^{\langle 11 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 12 \rangle}(\mathbf{p}) \\ \mathbf{A}^{\langle 21 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 22 \rangle}(\mathbf{p}) \end{bmatrix}$$

Uncertain Cooperative System Model (1)

Block-wise definition of the system matrix

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} \mathbf{A}^{\langle 11\rangle}(\mathbf{p}) & \mathbf{A}^{\langle 12\rangle}(\mathbf{p}) \\ \mathbf{A}^{\langle 21\rangle}(\mathbf{p}) & \mathbf{A}^{\langle 22\rangle}(\mathbf{p}) \end{bmatrix}$$

Example for the parameter-dependent matrix entries

$$a_{i,j}^{\langle 11\rangle}(\mathbf{p}) = \begin{cases} \frac{p_1}{c_i \cdot m_i} < 0 & \text{for } i = j = 1 \text{ and } i = j = \tilde{N} \\ \frac{p_2}{c_i \cdot m_i} > 0 & \text{for } i = j - 1\,,\ j \in \{2, \dots, \tilde{N}\} \\ \frac{p_2}{c_i \cdot m_i} > 0 & \text{for } i = j + 1\,,\ j \in \{1, \dots, \tilde{N} - 1\} \\ \frac{p_3}{c_i \cdot m_i} < 0 & \text{for } i = j\,,\ j \in \{2, \dots, \tilde{N}\} \\ 0 & \text{else} \end{cases}$$

Uncertain Cooperative System Model (2)

Block-wise definition of the system matrix

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} \mathbf{A}^{\langle 11 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 12 \rangle}(\mathbf{p}) \\ \mathbf{A}^{\langle 21 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 22 \rangle}(\mathbf{p}) \end{bmatrix}$$

Parameterization of the lower bounding system

$$\underline{\mathbf{p}} = \begin{bmatrix} -\left(\overline{\lambda}_{\mathrm{R}} \cdot \frac{A_{\mathrm{c}}}{l_{\mathrm{s}}} + (\overline{\alpha}_{\mathrm{B}} + \overline{\alpha}) \cdot A_{\mathrm{s}}\right) \\ \underline{\lambda}_{\mathrm{R}} \cdot \frac{A_{\mathrm{c}}}{l_{\mathrm{s}}} \\ -\left(2\overline{\lambda}_{\mathrm{R}} \cdot \frac{A_{\mathrm{c}}}{l_{\mathrm{s}}} + (\overline{\alpha}_{\mathrm{B}} + \overline{\alpha}) \cdot A_{\mathrm{s}}\right) \\ \underline{\alpha} \cdot A_{\mathrm{s}} \\ -\left(\overline{\alpha}_{\mathrm{T}} + \overline{\alpha} + \overline{\Delta\alpha} \cdot \delta_{\tilde{N}+i,\tilde{N}+1}\right) \cdot A_{\mathrm{s}} \\ \left(\frac{m_{\mathrm{a}}}{\tilde{N}} \cdot \left(1 + \delta_{\tilde{N}+i,2\tilde{N}} \cdot \overline{\Delta m}_{\mathrm{a}}\right)\right)^{-1} \end{bmatrix}$$

Uncertain Cooperative System Model (3)

Block-wise definition of the system matrix

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} \mathbf{A}^{\langle 11 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 12 \rangle}(\mathbf{p}) \\ \mathbf{A}^{\langle 21 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 22 \rangle}(\mathbf{p}) \end{bmatrix}$$

Parameterization of the upper bounding system

$$\overline{\mathbf{p}} = \begin{bmatrix} -\left(\underline{\lambda}_{\mathrm{R}} \cdot \frac{A_{\mathrm{c}}}{l_{\mathrm{s}}} + (\underline{\alpha}_{\mathrm{B}} + \underline{\alpha}) \cdot A_{\mathrm{s}}\right) \\ \overline{\lambda}_{\mathrm{R}} \cdot \frac{A_{\mathrm{c}}}{l_{\mathrm{s}}} \\ -\left(2\underline{\lambda}_{\mathrm{R}} \cdot \frac{A_{\mathrm{c}}}{l_{\mathrm{s}}} + (\underline{\alpha}_{\mathrm{B}} + \underline{\alpha}) \cdot A_{\mathrm{s}}\right) \\ \overline{\alpha} \cdot A_{\mathrm{s}} \\ -\left(\underline{\alpha}_{\mathrm{T}} + \underline{\alpha} + \underline{\Delta\alpha} \cdot \delta_{\tilde{N}+i,\tilde{N}+1}\right) \cdot A_{\mathrm{s}} \\ \left(\frac{m_{\mathrm{a}}}{\tilde{N}} \cdot \left(1 + \delta_{\tilde{N}+i,2\tilde{N}} \cdot \underline{\Delta m}_{\mathrm{a}}\right)\right)^{-1} \end{bmatrix}$$

Uncertain Cooperative System Model (4)

Block-wise definition of the system matrix

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} \mathbf{A}^{\langle 11 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 12 \rangle}(\mathbf{p}) \\ \mathbf{A}^{\langle 21 \rangle}(\mathbf{p}) & \mathbf{A}^{\langle 22 \rangle}(\mathbf{p}) \end{bmatrix}$$

Example for the sign pattern in the system matrix $\mathbf{A}(\mathbf{p})$ for $M = 0$

$$\mathbf{A}(\mathbf{p}) = \left[\begin{array}{cccc:cccc} - & + & 0 & 0 & + & 0 & 0 & 0 \\ + & - & + & 0 & 0 & + & 0 & 0 \\ 0 & + & - & + & 0 & 0 & + & 0 \\ 0 & 0 & + & - & 0 & 0 & 0 & + \\ \hdashline + & 0 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & + & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & + & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & 0 & - \end{array}\right]$$

Interval Observer Design

Decoupled bounding systems

$$\mathbf{A}_{\mathrm{O}}(\underline{\mathbf{p}})\cdot\hat{\mathbf{v}}(t)+\mathbf{B}\cdot\mathbf{u}(t)+\mathbf{H}\underline{\mathbf{y}}_{\mathrm{m}}(t)\leq\dot{\mathbf{x}}(t)\leq\mathbf{A}_{\mathrm{O}}(\overline{\mathbf{p}})\cdot\hat{\mathbf{w}}(t)+\mathbf{B}\cdot\mathbf{u}(t)+\mathbf{H}\overline{\mathbf{y}}_{\mathrm{m}}(t)$$

with the observer system matrix

$$\mathbf{A}_{\mathrm{O}}(\mathbf{p})=\mathbf{A}\left(\mathbf{p}\right)-\mathbf{HC}=\mathbf{A}\left(\mathbf{p}\right)-\boldsymbol{\mathcal{H}}$$

Requirements for admissible observer parameterizations

1. Guarantee of asymptotic stability
2. Preservation of cooperativity
3. Robustness and optimality?

Interval Observer Design

Decoupled bounding systems

$$\mathbf{A}_\mathrm{O}\left(\underline{\mathbf{p}}\right)\cdot\hat{\mathbf{v}}(t)+\mathbf{B}\cdot\mathbf{u}(t)+\mathbf{H}\underline{\mathbf{y}}_\mathrm{m}(t)\leq\dot{\hat{\mathbf{x}}}(t)\leq\mathbf{A}_\mathrm{O}(\overline{\mathbf{p}})\cdot\hat{\mathbf{w}}(t)+\mathbf{B}\cdot\mathbf{u}(t)+\mathbf{H}\overline{\mathbf{y}}_\mathrm{m}(t)$$

with the observer system matrix

$$\mathbf{A}_\mathrm{O}(\mathbf{p})=\mathbf{A}\left(\mathbf{p}\right)-\mathbf{HC}=\mathbf{A}\left(\mathbf{p}\right)-\boldsymbol{\mathcal{H}}$$

Guaranteed stabilizing, cooperativity preserving parameterization

$$\mathbf{H}=\kappa\mathbf{C}^T \quad \text{with} \quad \kappa>0$$

leading to

$$\boldsymbol{\mathcal{H}}=\mathbf{HC}=\kappa\mathbf{C}^T\mathbf{C} \quad \text{with} \quad \mathcal{H}_{i,j}=\begin{cases}\kappa & \text{for } i=j=(\xi\cdot(2M+1)-M)\ , \\ & \qquad \xi\in\{1,\ldots,N\} \\ \kappa & \text{for } i=j\ ,\ j\in\{\tilde{N}+1,2\tilde{N}\} \\ 0 & \text{else}\end{cases}$$

Interval Observer Design

Decoupled bounding systems

$$\mathbf{A}_{\mathrm{O}}(\underline{\mathbf{p}})\cdot\hat{\mathbf{v}}(t)+\mathbf{B}\cdot\mathbf{u}(t)+\mathbf{H}\underline{\mathbf{y}}_{\mathrm{m}}(t)\leq\dot{\hat{\mathbf{x}}}(t)\leq\mathbf{A}_{\mathrm{O}}(\overline{\mathbf{p}})\cdot\hat{\mathbf{w}}(t)+\mathbf{B}\cdot\mathbf{u}(t)+\mathbf{H}\overline{\mathbf{y}}_{\mathrm{m}}(t)$$

with the observer system matrix

$$\mathbf{A}_{\mathrm{O}}(\mathbf{p})=\mathbf{A}\left(\mathbf{p}\right)-\mathbf{HC}=\mathbf{A}\left(\mathbf{p}\right)-\boldsymbol{\mathcal{H}}$$

Note

- Advantage: Trivial stability proof by means of the Gershgorin circle theorem
- However: What is the optimal choice for the parameter κ?
- Are there other, not straightforward, parameterizations that may lead to better performance?

Optimization of the Observer Gain Matrix (1)

Gain matrix $\mathbf{H}$ without any structural restrictions except for cooperativity

- Definition of the error vector between the estimated and true lower and upper state bounds

$$\mathbf{e} = \begin{bmatrix} (\hat{\mathbf{v}} - \mathbf{v})^T & (\hat{\mathbf{w}} - \mathbf{w})^T \end{bmatrix}^T$$

- Error dynamics

$$\dot{\mathbf{e}} = \begin{bmatrix} \mathbf{A}(\underline{\mathbf{p}}) - \mathbf{HC} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}(\overline{\mathbf{p}}) - \mathbf{HC} \end{bmatrix} \mathbf{e} + \begin{bmatrix} \mathbf{H} \\ \mathbf{H} \end{bmatrix} \zeta$$

Optimization of the Observer Gain Matrix (1)

Gain matrix $\mathbf{H}$ without any structural restrictions except for cooperativity

- Definition of the error vector between the estimated and true lower and upper state bounds

$$\mathbf{e} = \begin{bmatrix} (\hat{\mathbf{v}} - \mathbf{v})^T & (\hat{\mathbf{w}} - \mathbf{w})^T \end{bmatrix}^T$$

- Augmented system output

$$\begin{aligned} \mathbf{y}_\infty &= \begin{bmatrix} \mathbf{0}_{(N+2)\times\tilde{N}} & \mathbf{0}_{(N+2)\times\tilde{N}} \\ -\nu \cdot \mathbf{I}_{\tilde{N}\times\tilde{N}} & \nu \cdot \mathbf{I}_{\tilde{N}\times\tilde{N}} \end{bmatrix} \mathbf{e} + \begin{bmatrix} \mathbf{I}_{(N+2)\times(N+2)} \\ \mathbf{0}_{\tilde{N}\times(N+2)} \end{bmatrix} \zeta \\ &= \mathbf{C}_\infty \mathbf{e} + \mathbf{D}_{\infty 1} \zeta \end{aligned}$$

$\Longrightarrow$ Comparison of the measurement errors ζ with the weighted diameter $(\hat{\mathbf{w}} - \mathbf{w}) - (\hat{\mathbf{v}} - \mathbf{v})$, corresponding parameter $\nu > 0$

Optimization of the Observer Gain Matrix (1)

Gain matrix $\mathbf{H}$ without any structural restrictions except for cooperativity

- LMI-based optimization problem with

$$\boldsymbol{\mathcal{L}}\left(\boldsymbol{\Theta}\right) := \begin{bmatrix} \boldsymbol{\Theta} & \mathbf{H} & \mathbf{P}_{\mathrm{O}}\mathbf{C}_{\infty}^{T} \\ \mathbf{H}^{T} & -\mathbf{I} & \mathbf{D}_{\infty 1}^{T} \\ \mathbf{C}_{\infty}\mathbf{P}_{\mathrm{O}} & \mathbf{D}_{\infty 1} & -\gamma_{\infty,\mathrm{O}}^{2}\mathbf{I} \end{bmatrix} \prec 0 \quad \text{with} \quad \boldsymbol{\Theta} \in \{\underline{\boldsymbol{\Theta}}, \overline{\boldsymbol{\Theta}}\}$$

  for the two extremal systems

$$\underline{\boldsymbol{\Theta}} := \mathbf{A}_{\mathrm{O}}(\underline{\mathbf{p}}) \cdot \mathbf{P}_{\mathrm{O}} + \mathbf{P}_{\mathrm{O}} \cdot \mathbf{A}_{\mathrm{O}}^{T}(\underline{\mathbf{p}})$$
$$\overline{\boldsymbol{\Theta}} := \mathbf{A}_{\mathrm{O}}(\overline{\mathbf{p}}) \cdot \mathbf{P}_{\mathrm{O}} + \mathbf{P}_{\mathrm{O}} \cdot \mathbf{A}_{\mathrm{O}}^{T}(\overline{\mathbf{p}})$$

- Lyapunov function candidate to ensure robust stability $\mathbf{P}_{\mathrm{O}} = \mathbf{P}_{\mathrm{O}}^{T} \succ 0$
- H_{∞} optimization problem by minimization of $\gamma_{\infty,\mathrm{O}} > 0$

Optimization of the Observer Gain Matrix (1)

Gain matrix $\mathbf{H}$ without any structural restrictions except for cooperativity

- Linearizing change of variables

$$\mathbf{Q}_{\mathrm{O}} = \mathbf{Q}_{\mathrm{O}}^T = \mathbf{P}_{\mathrm{O}}^{-1} \succ 0 \quad \text{with} \quad \mathbf{Y}_{\mathrm{O}}^T = \mathbf{Q}_{\mathrm{O}}\mathbf{H} = \mathbf{P}_{\mathrm{O}}^{-1}\mathbf{H}$$

- Minimization of $\mu_{\infty,\mathrm{O}} := \gamma_{\infty,\mathrm{O}}^2 > 0$ under consideration of

$$\boldsymbol{\mathcal{M}}(\boldsymbol{\Sigma}) := \begin{bmatrix} \boldsymbol{\Sigma} & \mathbf{Y}_{\mathrm{O}}^T & \mathbf{C}_{\infty}^T \\ \mathbf{Y}_{\mathrm{O}} & -\mathbf{I} & \mathbf{D}_{\infty 1}^T \\ \mathbf{C}_{\infty} & \mathbf{D}_{\infty 1} & -\mu_{\infty,\mathrm{O}}\mathbf{I} \end{bmatrix} \prec 0 \quad \text{with} \quad \boldsymbol{\Sigma} \in \{\underline{\boldsymbol{\Sigma}}, \overline{\boldsymbol{\Sigma}}\}$$

for the extremal systems

$$\underline{\boldsymbol{\Sigma}} := \mathbf{Q}_{\mathrm{O}}\mathbf{A}(\underline{\mathbf{p}}) - \mathbf{Y}_{\mathrm{O}}^T\mathbf{C} + \mathbf{A}^T(\underline{\mathbf{p}})\,\mathbf{Q}_{\mathrm{O}} - \mathbf{C}^T\mathbf{Y}_{\mathrm{O}}$$
$$\overline{\boldsymbol{\Sigma}} := \mathbf{Q}_{\mathrm{O}}\mathbf{A}(\overline{\mathbf{p}}) - \mathbf{Y}_{\mathrm{O}}^T\mathbf{C} + \mathbf{A}^T(\overline{\mathbf{p}})\,\mathbf{Q}_{\mathrm{O}} - \mathbf{C}^T\mathbf{Y}_{\mathrm{O}}$$

Optimization of the Observer Gain Matrix (1)

Gain matrix $\mathbf{H}$ without any structural restrictions except for cooperativity

- Iterative solution procedure ensuring cooperativity

$$\mathrm{col}\Big(\big(\mathbf{A}(\mathbf{p}) - \check{\mathbf{Q}}_{\mathrm{O}}^{-1}\mathbf{Y}_{\mathrm{O}}^{T}\mathbf{C}\big) \circ (\mathbf{E}-\mathbf{I})\Big) \geq \mathbf{0}$$

for both $\mathbf{p} \in \{\underline{\mathbf{p}}, \overline{\mathbf{p}}\}$ with $\circ$ denoting the element-wise defined Hadamard product of two matrices with identical dimensions

Note

In the case of a description of the feasible parameter domains by multiple subintervals $[\mathbf{p}_i]$, $i \in \{1, \ldots, L\}$, the complete state domains are described by the interval union over all respective interval-valued state estimates

Optimization of the Observer Gain Matrix (2)

Obviously cooperativity-preserving structure

$$\mathbf{H} = (\boldsymbol{\mathcal{K}}\mathbf{C})^T \quad \text{with} \quad \boldsymbol{\mathcal{K}} = \mathrm{diag}\left\{\boldsymbol{\kappa}\right\} \quad \text{and}$$

$$\boldsymbol{\kappa} = \begin{bmatrix}\kappa_1 & \dots & \kappa_{N+2}\end{bmatrix}\,,\ \kappa_i > 0\,,\ i \in \{1, \dots, N+2\}$$

Advantage

Reduced number of computational operations during the application of the observer due to the sparse structure of the gain matrix $\mathbf{H}$

Optimization of the Observer Gain Matrix (2)

Obviously cooperativity-preserving structure

$$\mathbf{H} = (\boldsymbol{\mathcal{K}}\mathbf{C})^T \quad \text{with} \quad \boldsymbol{\mathcal{K}} = \mathrm{diag}\{\boldsymbol{\kappa}\} \quad \text{and}$$

$$\boldsymbol{\kappa} = \begin{bmatrix} \kappa_1 & \dots & \kappa_{N+2} \end{bmatrix}, \ \kappa_i > 0\,, \ i \in \{1, \dots, N+2\}$$

Advantage

Reduced number of computational operations during the application of the observer due to the sparse structure of the gain matrix $\mathbf{H}$

Yet an even simpler version

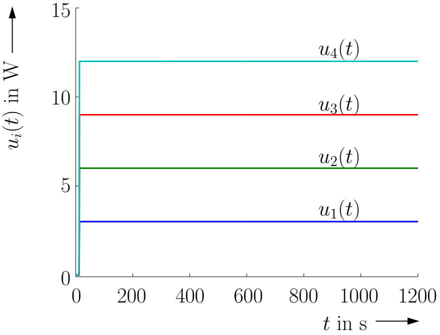
$$\mathbf{H} = (\boldsymbol{\mathcal{K}}\mathbf{C})^T \quad \text{with} \quad \boldsymbol{\mathcal{K}} = \mathrm{diag}\{\boldsymbol{\kappa}\} \quad \text{and}$$

$$\boldsymbol{\kappa} = \begin{bmatrix} \kappa_1 & \dots & \kappa_{N+2} \end{bmatrix}, \ \kappa_1 = \dots = \kappa_{N+2} > 0$$

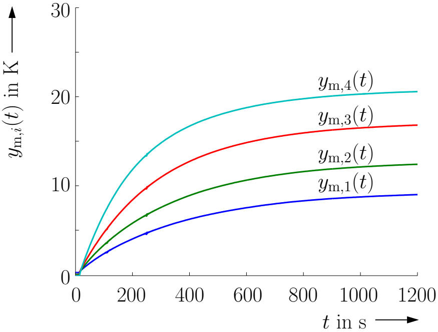
Experimental Results for Verified Parameter Identification

- Measurement uncertainty: $[-0.75\ ;\ 0.75]\ \mathrm{K}$
- Sampling time: $1\,\mathrm{s}$

Control signals $\mathbf{u}(t)$ of all Peltier elements



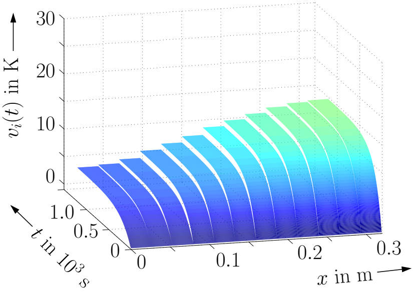
Measured rod temperatures (segment midpoints)



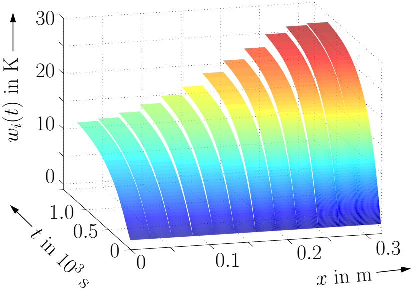
Experimental Results for Verified Parameter Identification

Offline parameter identification: Average diameter of the rod temperature intervals $6.86\,\mathrm{K}$, corresponding to $\mathbf{H} = \mathbf{0}$

Lower bounds $v_i(t)$



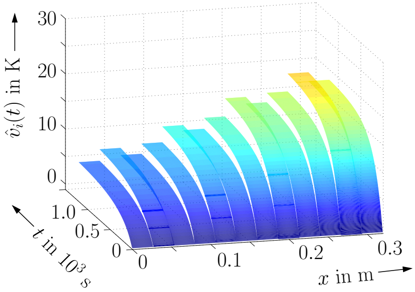
Upper bounds $w_i(t)$



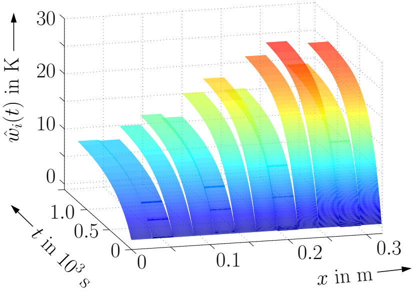
Experimental Results for the Observer Application

Online state observation: Average diameter of rod temperature intervals $3.90\,\mathrm{K}$, full gain matrix $\mathbf{H} \neq \mathbf{0}$

Lower bounds $\hat{v}_i(t)$

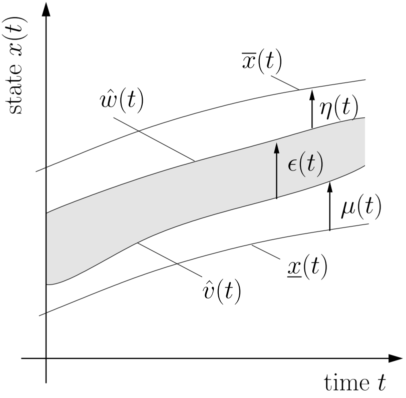


Upper bounds $\hat{w}_i(t)$



Cooperativity-Preserving Feedback Control (1)

- Worst-case lower and upper bounds $\underline{\mathbf{x}}(t)$ and $\overline{\mathbf{x}}(t)$ of the true states
- Worst-case, component-wise non-negative deviation $\boldsymbol{\mu}_i(t) = \hat{\mathbf{v}}_i(t) - \underline{\mathbf{x}}(t)$
- Component-wise defined non-negative interval diameters $\boldsymbol{\epsilon}_i(t) = \hat{\mathbf{w}}_i(t) - \hat{\mathbf{v}}_i(t)$ for the i-th observer
- Component-wise non-negative deviation $\boldsymbol{\eta}_i(t) = \overline{\mathbf{x}}(t) - \hat{\mathbf{w}}_i(t)$, $i \in \{1, \ldots, L\}$



Cooperativity-Preserving Feedback Control (2)

Definition of the control law

Generalization of the *classical* state feedback control approach

$$\mathbf{u}(t) = \mathbf{u}_{\mathrm{ff}}(t) - \sum_{\varsigma=1}^{L} \left(\underline{\mathbf{K}}_{\varsigma} \hat{\mathbf{v}}_{\varsigma}(t) + \overline{\mathbf{K}}_{\varsigma} \hat{\mathbf{w}}_{\varsigma}(t) \right)$$

Cooperativity-Preserving Feedback Control (2)

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Stability and optimality requirements

$$\dot{\boldsymbol{\zeta}}(t) = \begin{bmatrix} \underline{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{A}} \end{bmatrix} \cdot \boldsymbol{\zeta}(t) + L \cdot \begin{bmatrix} \overline{\mathbf{B}} \\ \underline{\mathbf{B}} \end{bmatrix} \cdot \mathbf{w}(t) + \begin{bmatrix} \overline{\mathbf{B}} \\ \underline{\mathbf{B}} \end{bmatrix} \cdot \boldsymbol{\nu}(t)$$

with

$$\boldsymbol{\nu}(t) = - \begin{bmatrix} \sum_{\varsigma=1}^{L} \underline{\mathbf{K}}_{\varsigma} & \sum_{\varsigma=1}^{L} \overline{\mathbf{K}}_{\varsigma} \end{bmatrix} \cdot \boldsymbol{\zeta}(t)$$

Cooperativity-Preserving Feedback Control (2)

Definition of the control law

Generalization of the *classical* state feedback control approach

$$\mathbf{u}(t) = \mathbf{u}_{\mathrm{ff}}(t) - \sum_{\varsigma=1}^{L} \left(\underline{\mathbf{K}}_{\varsigma} \hat{\mathbf{v}}_{\varsigma}(t) + \overline{\mathbf{K}}_{\varsigma} \hat{\mathbf{w}}_{\varsigma}(t) \right)$$

Stability and optimality requirements

$$\dot{\boldsymbol{\zeta}}(t) = \begin{bmatrix} \underline{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{A}} \end{bmatrix} \cdot \boldsymbol{\zeta}(t) + L \cdot \begin{bmatrix} \underline{\mathbf{B}} \\ \overline{\mathbf{B}} \end{bmatrix} \cdot \mathbf{w}(t) + \begin{bmatrix} \underline{\mathbf{B}} \\ \overline{\mathbf{B}} \end{bmatrix} \cdot \boldsymbol{\nu}(t)$$

with

$$\boldsymbol{\mathcal{A}} = \begin{bmatrix} \underline{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{A}} \end{bmatrix} , \quad \boldsymbol{\mathcal{B}}_1 = L \cdot \begin{bmatrix} \underline{\mathbf{B}} \\ \overline{\mathbf{B}} \end{bmatrix} , \quad \boldsymbol{\mathcal{B}}_2 = \begin{bmatrix} \underline{\mathbf{B}} \\ \overline{\mathbf{B}} \end{bmatrix}$$

Cooperativity-Preserving Feedback Control (3)

H_∞ performance specification

Augmented output equations

$$\mathbf{y}_\infty(t) = \begin{bmatrix} -\mathbf{\Xi I}_{n_x \times n_x} & \mathbf{\Xi I}_{n_x \times n_x} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \boldsymbol{\zeta}(t) + \begin{bmatrix} \mathbf{0} \\ L\mathbf{I}_{n_\nu} \\ \mathbf{0} \end{bmatrix} \cdot \mathbf{w}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{\Gamma I}_{n_\nu} \end{bmatrix} \cdot \boldsymbol{\nu}(t)$$

with $\mathbf{\Xi} = \mathrm{diag}\{\xi_j\}$, $\xi_j > 0$ and $\mathbf{\Gamma} = \mathrm{diag}\{\gamma_j\}$, $\gamma_j > 0$

Cooperativity-Preserving Feedback Control (3)

H_∞ performance specification

Augmented output equations

$$\mathbf{y}_\infty(t) = \begin{bmatrix} -\boldsymbol{\Xi}\mathbf{I}_{n_\mathrm{x}\times n_\mathrm{x}} & \boldsymbol{\Xi}\mathbf{I}_{n_\mathrm{x}\times n_\mathrm{x}} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \boldsymbol{\zeta}(t) + \begin{bmatrix} \mathbf{0} \\ L\mathbf{I}_{n_\nu} \\ \mathbf{0} \end{bmatrix} \cdot \mathbf{w}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{\Gamma}\mathbf{I}_{n_\nu} \end{bmatrix} \cdot \boldsymbol{\nu}(t)$$

with $\boldsymbol{\Xi} = \mathrm{diag}\left\{\xi_j\right\}$, $\xi_j > 0$ and $\boldsymbol{\Gamma} = \mathrm{diag}\left\{\gamma_j\right\}$, $\gamma_j > 0$

Abbreviations

$$\boldsymbol{\mathcal{C}}_\infty = \begin{bmatrix} -\boldsymbol{\Xi}\mathbf{I}_{n_\mathrm{x}\times n_\mathrm{x}} & \boldsymbol{\Xi}\mathbf{I}_{n_\mathrm{x}\times n_\mathrm{x}} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \boldsymbol{\mathcal{D}}_{\infty 1} = \begin{bmatrix} \mathbf{0} \\ L\mathbf{I}_{n_\nu\times n_\nu} \\ \mathbf{0} \end{bmatrix}, \boldsymbol{\mathcal{D}}_{\infty 2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{\Gamma}\mathbf{I}_{n_\nu\times n_\nu} \end{bmatrix}$$

Cooperativity-Preserving Feedback Control (4)

Design LMI: Stability and optimality

$$\begin{bmatrix} \boldsymbol{\mathcal{S}}_{1,1} & \boldsymbol{\mathcal{B}}_1 & \boldsymbol{\mathcal{S}}_{1,3} \\ \boldsymbol{\mathcal{B}}_1^T & -\gamma_\infty \mathbf{I} & \boldsymbol{\mathcal{D}}_{\infty 1}^T \\ \boldsymbol{\mathcal{S}}_{3,1} & \boldsymbol{\mathcal{D}}_{\infty 1} & -\gamma_\infty \mathbf{I} \end{bmatrix} \prec 0$$

with

$$\boldsymbol{\mathcal{S}}_{1,1} = (\mathbf{I}_{2\times 2} \otimes \mathbf{Q})\, \boldsymbol{\mathcal{A}}^T - \left[\sum_{\varsigma=1}^{L} \underline{\mathbf{Y}}_\varsigma \quad \sum_{\varsigma=1}^{L} \overline{\mathbf{Y}}_\varsigma \right]^T \cdot \boldsymbol{\mathcal{B}}_2^T$$

$$+ \underbrace{\boldsymbol{\mathcal{A}}\, (\mathbf{I}_{2\times 2} \otimes \mathbf{Q}) - \boldsymbol{\mathcal{B}}_2 \cdot \left[\sum_{\varsigma=1}^{L} \underline{\mathbf{Y}}_\varsigma \quad \sum_{\varsigma=1}^{L} \overline{\mathbf{Y}}_\varsigma \right]}_{\boldsymbol{\mathcal{A}}_{\mathrm{Q}}}$$

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Design LMI: Stability and optimality

$$\begin{bmatrix} \boldsymbol{\mathcal{S}}_{1,1} & \boldsymbol{\mathcal{B}}_1 & \boldsymbol{\mathcal{S}}_{1,3} \\ \boldsymbol{\mathcal{B}}_1^T & -\gamma_\infty \mathbf{I} & \boldsymbol{\mathcal{D}}_{\infty 1}^T \\ \boldsymbol{\mathcal{S}}_{3,1} & \boldsymbol{\mathcal{D}}_{\infty 1} & -\gamma_\infty \mathbf{I} \end{bmatrix} \prec 0$$

with

$$\boldsymbol{\mathcal{S}}_{1,3} = \boldsymbol{\mathcal{S}}_{3,1}^T = (\mathbf{I}_{2\times 2} \otimes \mathbf{Q})\, \boldsymbol{\mathcal{C}}_\infty^T - \begin{bmatrix} \sum_{\varsigma=1}^{L} \underline{\mathbf{Y}}_\varsigma & \sum_{\varsigma=1}^{L} \overline{\mathbf{Y}}_\varsigma \end{bmatrix}^T \cdot \boldsymbol{\mathcal{D}}_{\infty 2}^T$$

Cooperativity-Preserving Feedback Control (5)

Summarized in an augmented set of state equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\underline{\mathbf{x}}(t)\\ \boldsymbol{\mu}_1(t)\\ \vdots\\ \boldsymbol{\mu}_L(t)\\ \boldsymbol{\epsilon}_1(t)\\ \vdots\\ \boldsymbol{\epsilon}_L(t)\\ \overline{\mathbf{x}}(t)\end{bmatrix} = \underbrace{\left(\begin{bmatrix}\mathbb{A}_{1,1} & \mathbf{0}\\ \mathbf{0} & \mathbb{A}_{2,2}\end{bmatrix} - \begin{bmatrix}\mathbb{B}_{1,1} & \mathbf{0}\\ \mathbf{0} & \mathbb{B}_{2,2}\end{bmatrix}\cdot\begin{bmatrix}\mathbb{K}_{1,1} & \mathbf{0}\\ \mathbf{0} & \mathbb{K}_{2,2}\end{bmatrix}\right)}_{=\mathbb{A}_{\mathrm{K}}\left(\underline{\mathbf{K}}_{\varsigma},\overline{\mathbf{K}}_{\varsigma}\right)}\cdot\begin{bmatrix}\underline{\mathbf{x}}(t)\\ \boldsymbol{\mu}_1(t)\\ \vdots\\ \boldsymbol{\mu}_L(t)\\ \boldsymbol{\epsilon}_1(t)\\ \vdots\\ \boldsymbol{\epsilon}_L(t)\\ \overline{\mathbf{x}}(t)\end{bmatrix} + \tilde{\mathbb{S}}\cdot\begin{bmatrix}\mathbf{y}_{\mathrm{d}}(t)\\ \mathbf{y}_{\mathrm{m}}(t)\end{bmatrix}$$

Cooperativity-Preserving Feedback Control (6)

Generalization for preservation of cooperativity

- Use of
$$\mathbb{A}_{\mathrm{Q}} = \mathbb{A}_{\mathrm{K}}\left(\underline{\mathbf{K}}_{\varsigma}, \overline{\mathbf{K}}_{\varsigma}\right) \cdot \left(\mathbf{I}_{(2L+2)\times(2L+2)} \otimes \mathbf{Q}\right)$$
instead of $\boldsymbol{\mathcal{A}}_{\mathrm{Q}}$
- Subsequent iterative computation of the controller gains to ensure cooperativity of the augmented system model as well as stability with a parameter-independent Lyapunov function approach
$\Longrightarrow$ Generalization of the iteration described in detail for the observer parameterization
- Simultaneous minimization of $\gamma_{\infty} > 0$

Conclusions and Outlook on Future Work

- Cooperative system models (typical for thermal and fluidic systems, compartment models in biology and medicine, probabilities in Markov chain models)
- Design of interval observers for computation of guaranteed state bounds on the basis of uncertain measurements
- Optimization of the observer gain under the constraints of asymptotic stability and cooperativity
- Exploitation of the duality principle: Cooperativity-preserving observer-based state feedback control

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- Cooperative system models (typical for thermal and fluidic systems, compartment models in biology and medicine, probabilities in Markov chain models)
- Design of interval observers for computation of guaranteed state bounds on the basis of uncertain measurements
- Optimization of the observer gain under the constraints of asymptotic stability and cooperativity
- Exploitation of the duality principle: Cooperativity-preserving observer-based state feedback control
- Experimental validation for further application scenarios, e.g., temperature control for Solid Oxide Fuel Cells
- State feedback vs. output feedback control

Vielen Dank für Ihre Aufmerksamkeit!

Thank you for your attention!

Спасибо за Ваше внимание!

Merci beaucoup pour votre attention!

Dziękuję bardzo za uwagę!

¡Muchas gracias por su atención!

Grazie mille per la vostra attenzione!