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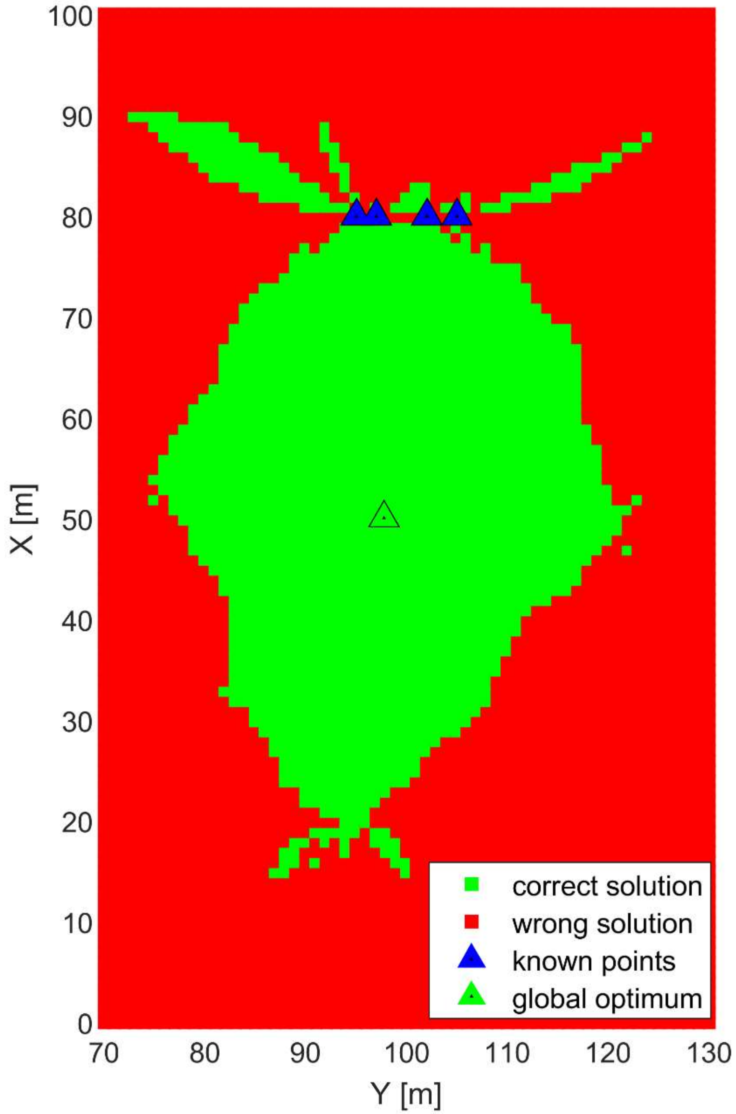
Global Optimisation as a Geodetic Tool in Network Adjustment

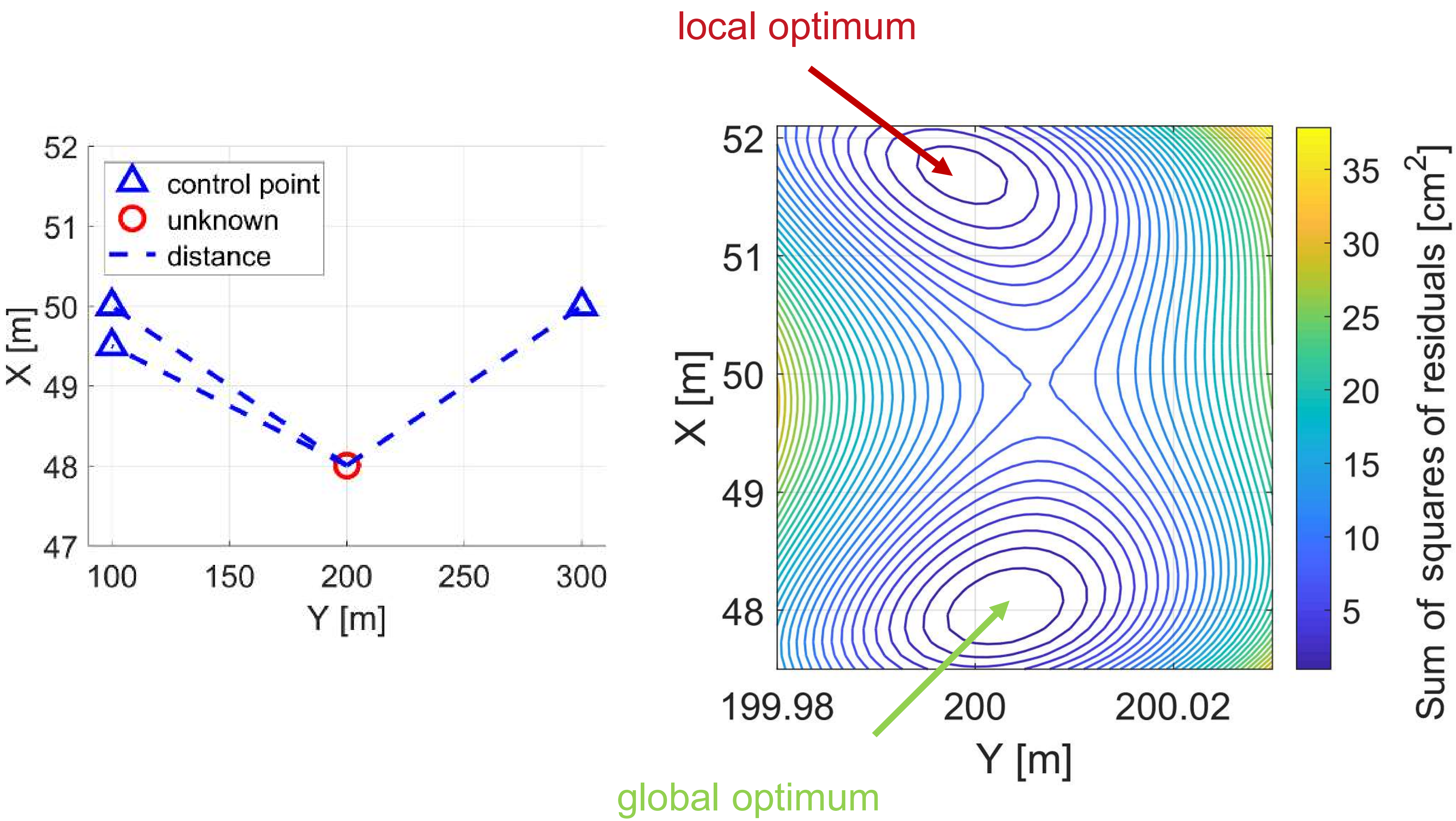
11th Summer Workshop on Interval Methods, July 2018

Tomke Lambertus & Jörg Reinking

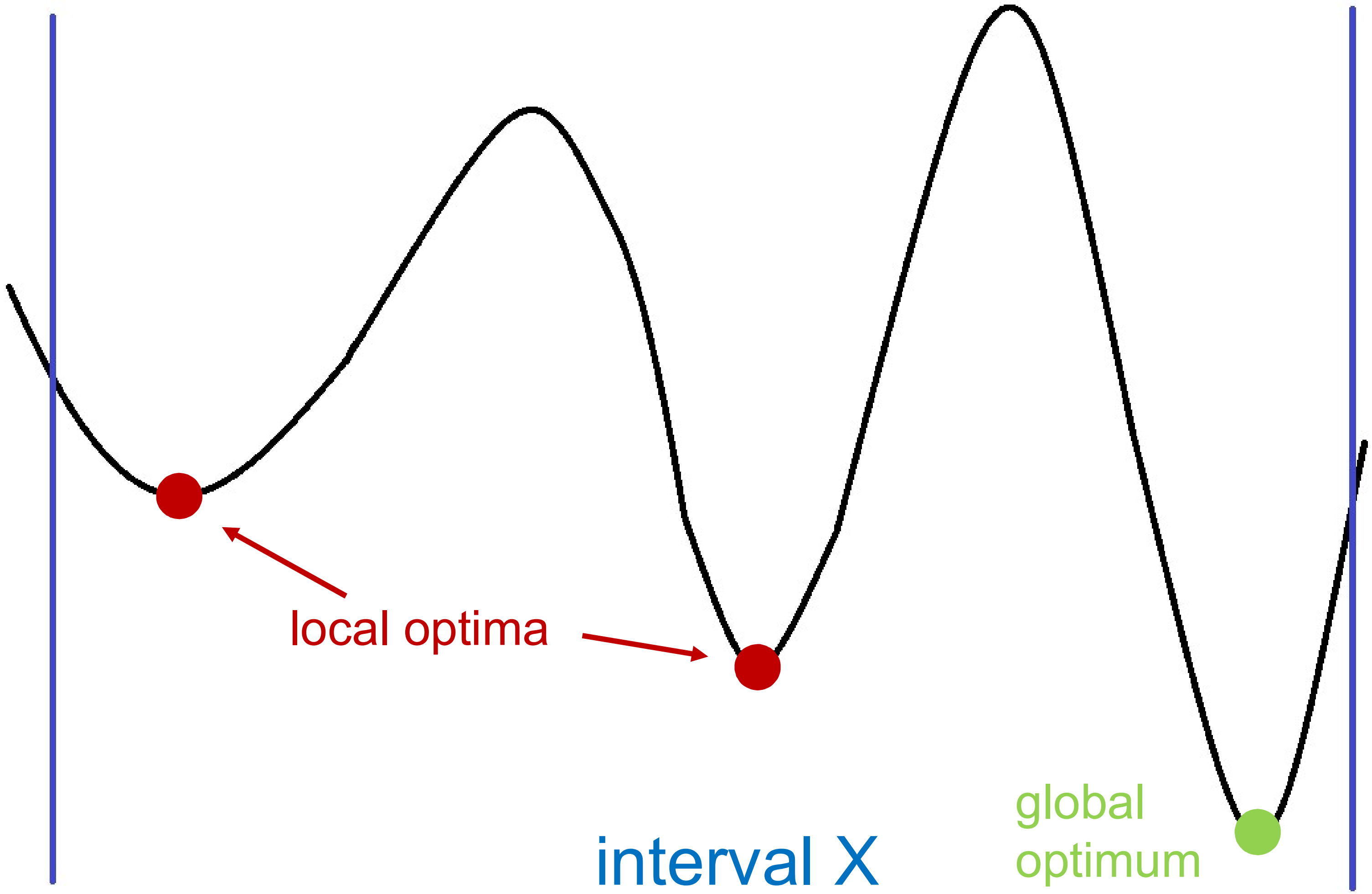
EXAMPLE RESECTION



EXAMPLE INTERSECTION



INTERVAL-BASED GLOBAL OPTIMISATION



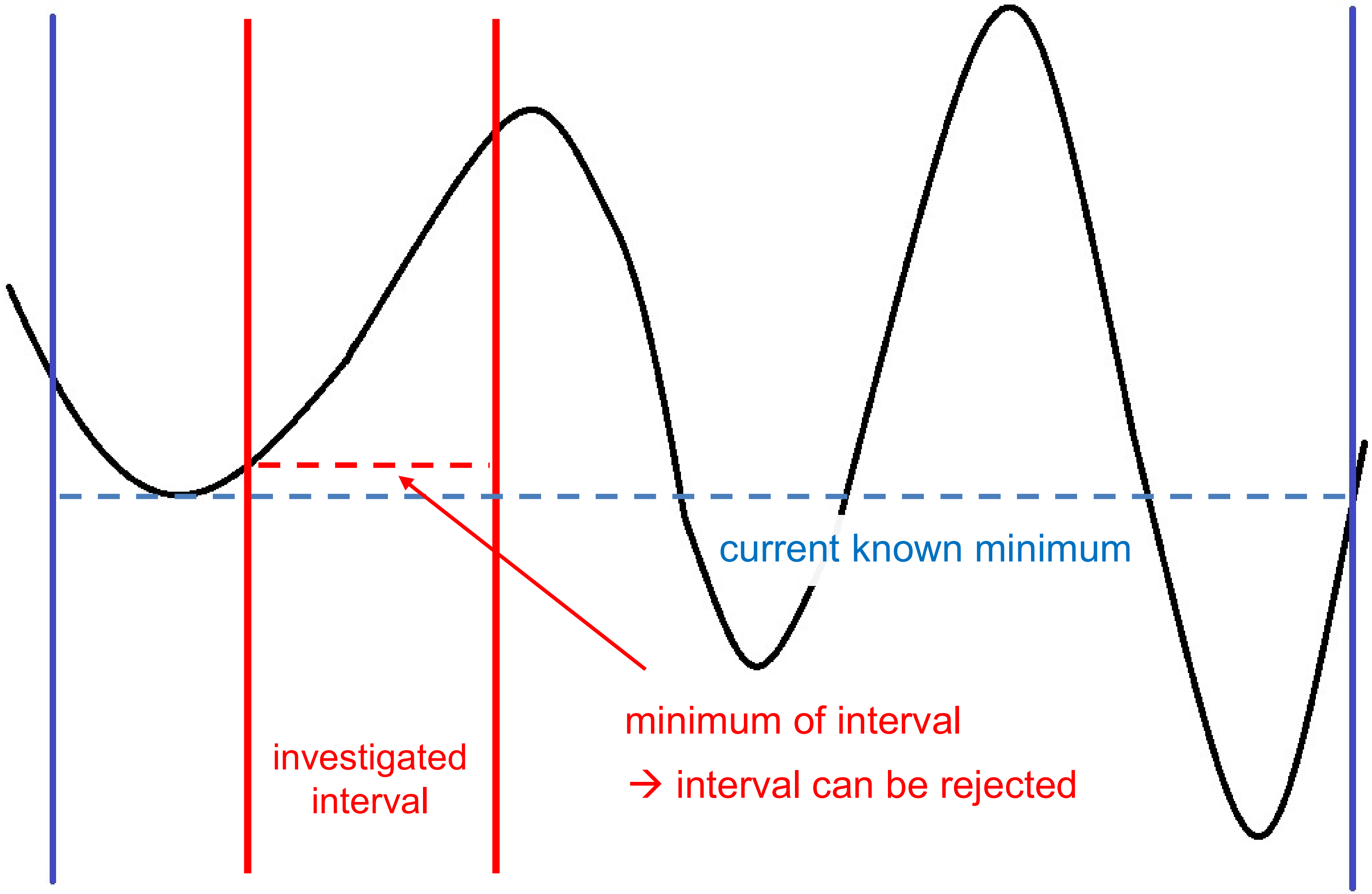
BRANCH-AND-BOUND-STRATEGY

- Start: Definition initial start interval

Loop:

- Choose interval of interest in list of intervals
- Bounding
  - Cut off test

CUT-OFF TEST


current known minimum
minimum of interval
→ interval can be rejected
investigated
interval

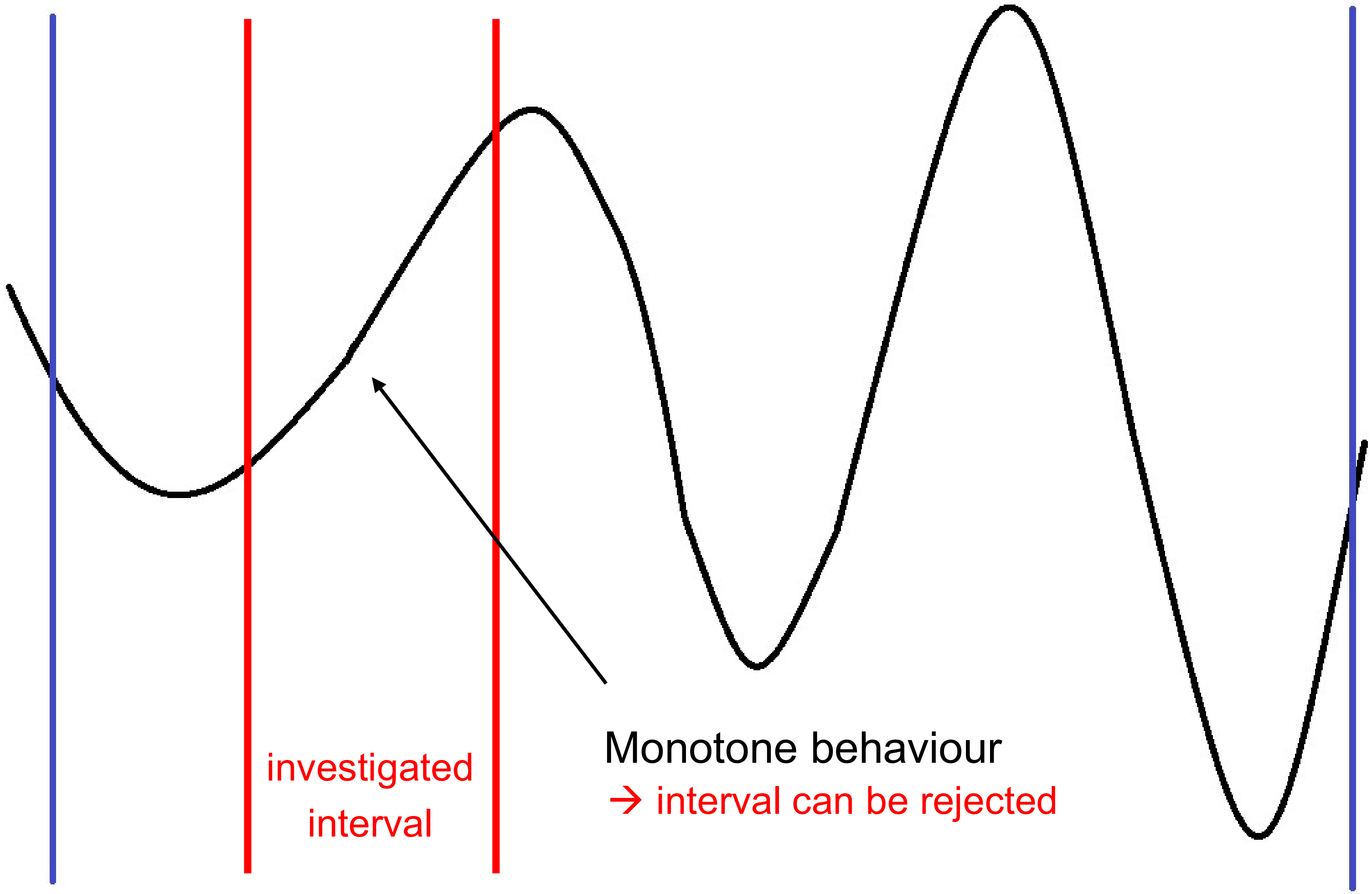
BRANCH-AND-BOUND-STRATEGY

- Start: Definition initial start interval

Loop:

- Choose interval of interest in list of intervals
- Bounding
  - Cut off test
  - Monotonicity test

MONOTONICITY TEST



BRANCH-AND-BOUND-STRATEGY

- Start: Definition initial start interval

Loop:

- Choose interval of interest in list of intervals
- Bounding
  - Cut off test
  - Monotonicity test
  - Interval-Newton-method

- Interval version from the Newton method
- Guarantees to find all roots in an interval function

mid point

function value of mid point

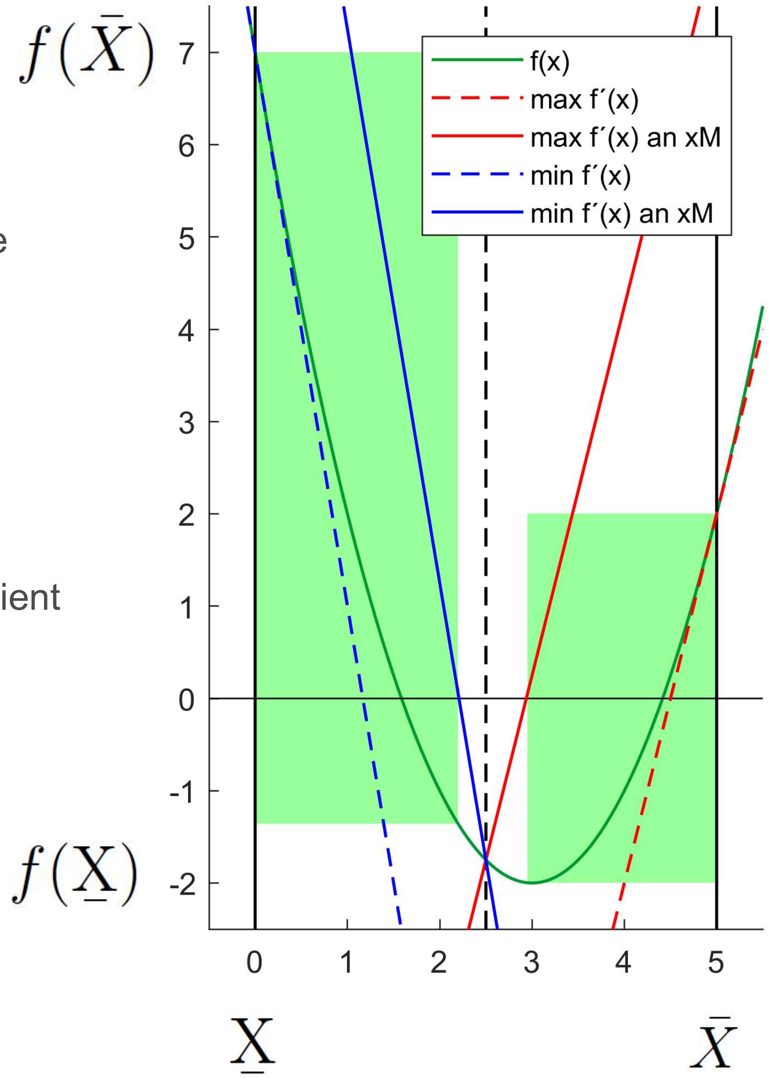
$$N(x_{M_{k+1}}, X_{k+1}) = X_k \cap x_{M_k} - \frac{f(x_{M_k})}{f'(X_k)}$$

current interval

max & min gradient

- Example

$$F(X) = (X-3)^2 - 2$$
$$X = [0, 5]$$



→ yield to one, two or an empty interval

BRANCH-AND-BOUND-STRATEGY

- Start: Definition initial start interval

Loop:

- Choose interval of interest in list of intervals
- Bounding
  - Cut off test
  - Monotonicity test
  - Interval-Newton-method
- Branching
  - Subdividing an interval

- Stop criteria: when the global optimum is found and/or list of intervals is empty

COST FUNCTION – EXAMPLE RESECTION

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- Cost function: $v^T P v$ → Sum of squares of weighted residuals
  v … vector of residuals
  P … weight matrix

$$v = arctan\left(\frac{y_{known} - \boxed{Y_{ship}}}{x_{known} - \boxed{X_{ship}}}\right) + o - measurement$$

intervals for optimisation

- 1.-3. Derivatives:

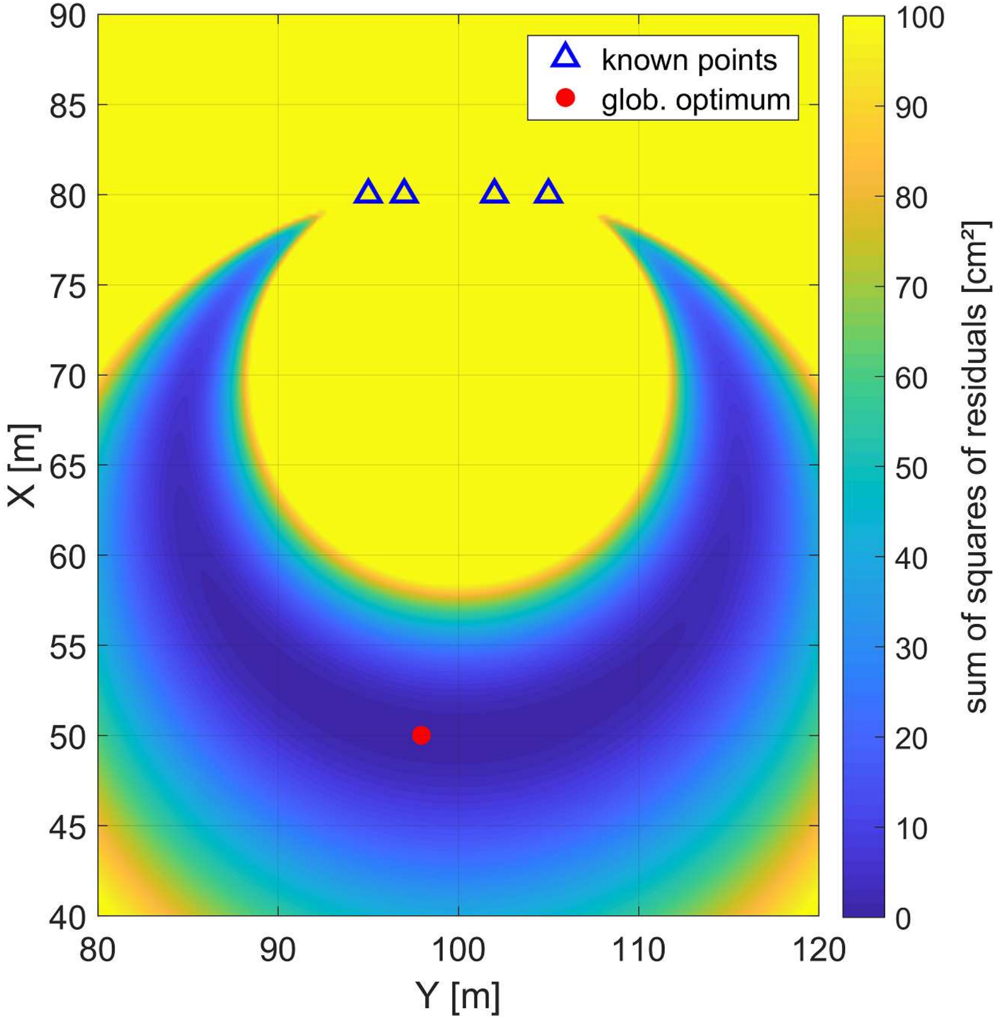
$$\frac{\partial v^T P v}{\partial x_i} = \frac{\partial v^T}{\partial x_i} P v + v^T P \frac{\partial v}{\partial x_i}$$

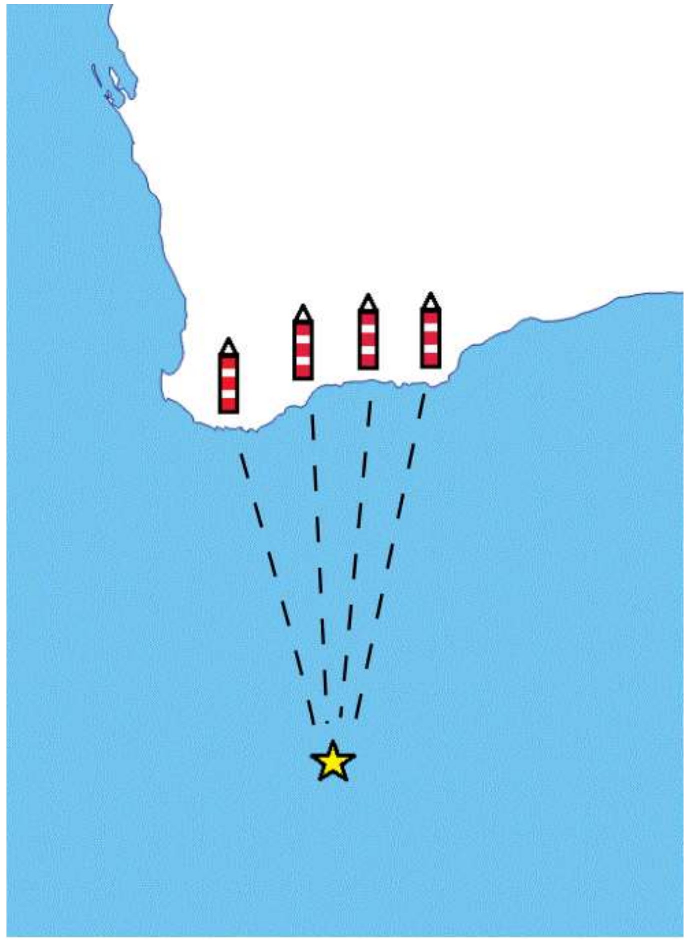
$$\frac{\partial^2 v^T P v}{\partial x_i \partial x_j} = \frac{\partial^2 v^T}{\partial x_i \partial x_j} P v + \frac{\partial v^T}{\partial x_i} P \frac{\partial v}{\partial x_j} + \frac{\partial v^T}{\partial x_j} P \frac{\partial v}{\partial x_i} + v^T P \frac{\partial^2 v}{\partial x_i \partial x_j}$$

$$\frac{\partial^3 v^T P v}{\partial x_i \partial x_j \partial x_k} = \frac{\partial^3 v^T}{\partial x_i \partial x_j \partial x_k} P v + \frac{\partial^2 v^T}{\partial x_i \partial x_j} P \frac{\partial v}{\partial x_k} + \frac{\partial^2 v^T}{\partial x_i \partial x_k} P \frac{\partial v}{\partial x_j} + \frac{\partial v^T}{\partial x_i} P \frac{\partial^2 v}{\partial x_j \partial x_k}$$
$$+ \frac{\partial^2 v^T}{\partial x_j \partial x_k} P \frac{\partial v}{\partial x_i} + \frac{\partial v^T}{\partial x_j} P \frac{\partial^2 v}{\partial x_i \partial x_k} + \frac{\partial v^T}{\partial x_k} P \frac{\partial^2 v}{\partial x_i \partial x_j} + v^T P \frac{\partial^3 v}{\partial x_i \partial x_j \partial x_k}$$

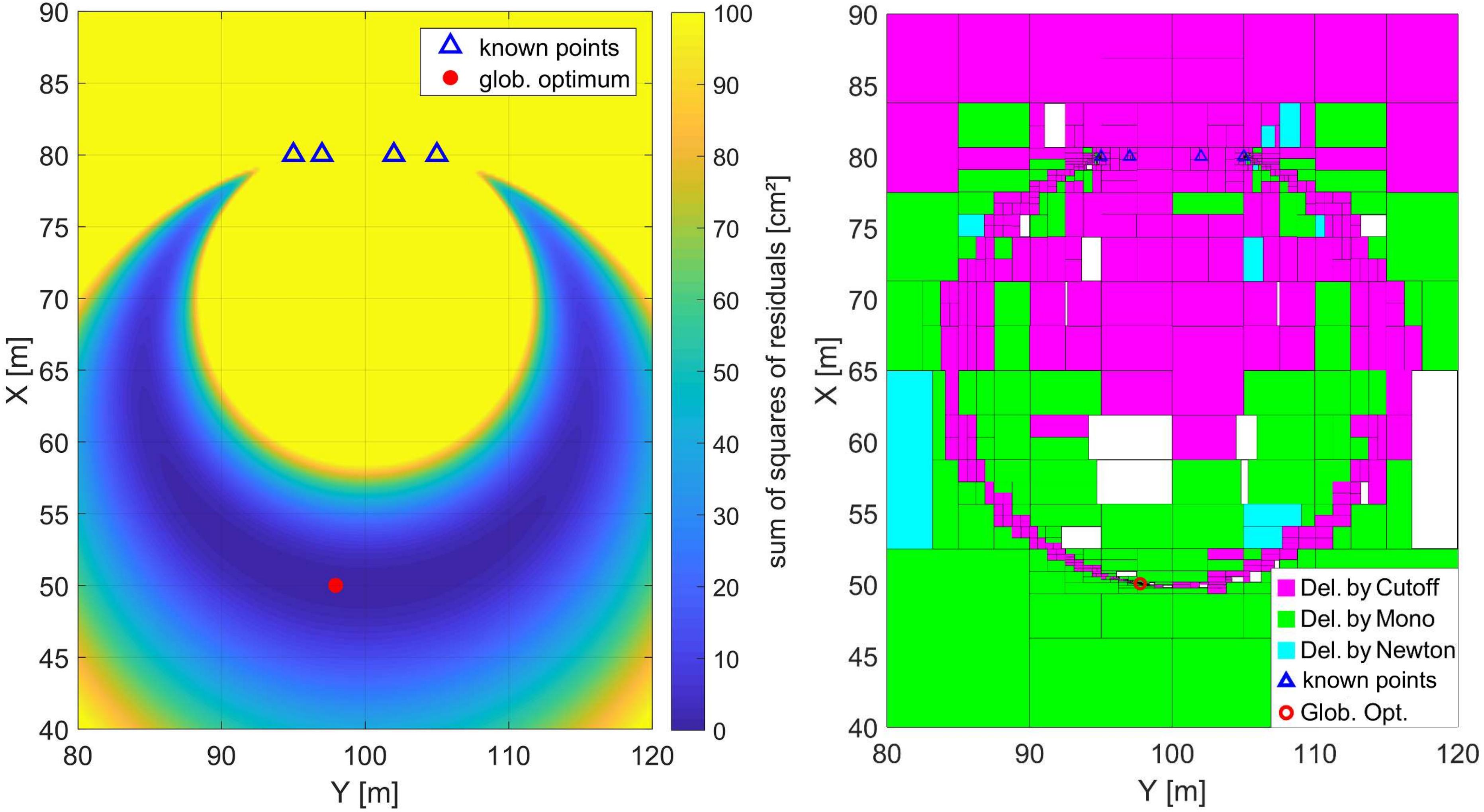
- Approximation of function intervals by Taylor expansion
  - Reduce the overestimation due to interval dependency

EXAMPLE RESECTION

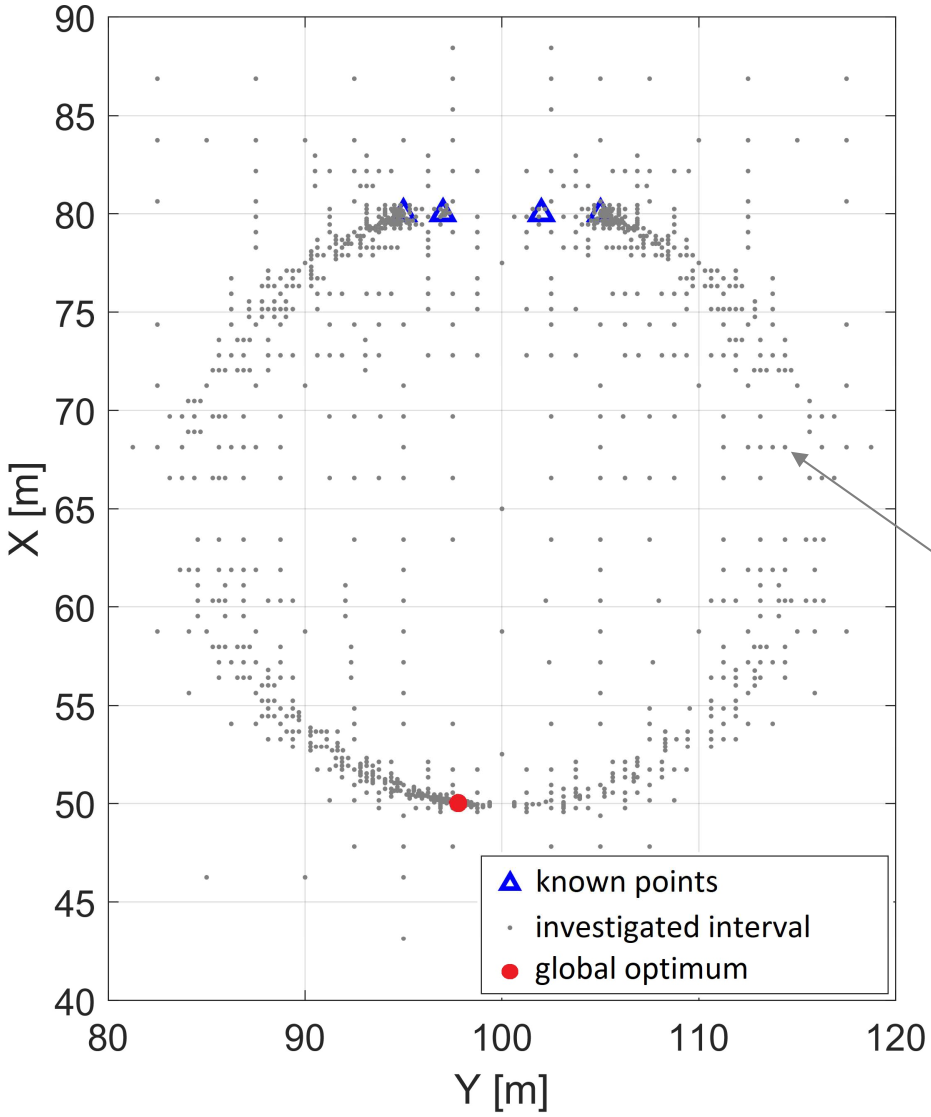




EXAMPLE RESECTION



EXAMPLE RESECTION – CLUSTER EFFECT



mid points of investigated intervals

CONCLUSION

- **Interval-based global optimisation** using **branch & bound - strategy** is a meaningful tool for problems having multimodal cost functions
- The correct solution of non-linear problems is found

Pros:

- The global optimisation yield the correct solution even if the classic non-linear adjustment (e.g. GMM) fails or leads to a local optimum only
- No requirement of adequate initial values
- Guarantee that the global optimum has been found if one exists

Cons:

- High computational effort
- The knowledge of high-order derivatives is useful but also costly
- Interval dependency leads to expanded interval bounds
- The computation time rapidly increases with the number of unknowns

Thanks for attention!

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