Global Optimisation as a Geodetic Tool in Network Adjustment

11th Summer Workshop on Interval Methods, July 2018

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EXAMPLE RESECTION
EXAMPLE INTERSECTION

- **Global optimum**
- **Local optimum**
INTERVAL-BASED GLOBAL OPTIMISATION

local optima

interval X

global optimum
BRANCH-AND-BOUND-STRATEGY

• Start: Definition initial start interval

Loop:
  • Choose interval of interest in list of intervals
  • Bounding
    • Cut off test
CUT-OFF TEST

investigated interval

minimum of interval
→ interval can be rejected

current known minimum
BRANCH-AND-BOUND-STRATEGY

- Start: Definition initial start interval

Loop:
- Choose interval of interest in list of intervals
- Bounding
  - Cut off test
  - Monotonicity test
MONOTONICITY TEST

investigated interval

Monotone behaviour
→ interval can be rejected
BRANCH-AND-BOUND-STRATEGY

- Start: Definition initial start interval

Loop:
- Choose interval of interest in list of intervals
- Bounding
  - Cut off test
  - Monotonicity test
  - Interval-Newton-method
INTERVAL-NEWTON-METHOD

- Interval version from the Newton method
- Guarantees to find all roots in an interval function

\[ N(x_{M_{k+1}}, X_{k+1}) = X_k \cap x_{M_k} - \frac{f(x_{M_k})}{f'(X_k)} \]

- Example

\[ F(X) = (X - 3)^2 - 2 \]

\[ X = [0, 5] \]

\[ \Rightarrow \text{yield to one, two or an empty interval} \]
BRANCH-AND-BOUND-STRATEGY

• Start: Definition initial start interval

Loop:
  • Choose interval of interest in list of intervals
  • Bounding
    • Cut off test
    • Monotonicity test
    • Interval-Newton-method
  • Branching
    • Subdividing an interval

• Stop criteria: when the global optimum is found and/or list of intervals is empty
COST FUNCTION – EXAMPLE RESECTION

- Cost function: \( v^T P v \rightarrow \) Sum of squares of weighted residuals
  
  \( v \) … vector of residuals
  
  \( P \) … weight matrix

  \[ v = \arctan \left( \frac{y_{\text{known}}}{x_{\text{known}}} - \frac{Y_{\text{ship}}}{X_{\text{ship}}} \right) + o - \text{measurement} \]

- 1.-3. Derivatives:

  \[ \frac{\partial v^T P v}{\partial x_i} = \frac{\partial v^T}{\partial x_i} P v + v^T P \frac{\partial v}{\partial x_i} \]

  \[ \frac{\partial^2 v^T P v}{\partial x_i \partial x_j} = \frac{\partial^2 v^T}{\partial x_i \partial x_j} P v + \frac{\partial v^T}{\partial x_i} P \frac{\partial v}{\partial x_j} + \frac{\partial v^T}{\partial x_j} P \frac{\partial v}{\partial x_i} + v^T P \frac{\partial^2 v}{\partial x_i \partial x_j} \]

  \[ \frac{\partial^3 v^T P v}{\partial x_i \partial x_j \partial x_k} = \frac{\partial^3 v^T}{\partial x_i \partial x_j \partial x_k} P v + \frac{\partial^2 v^T}{\partial x_i \partial x_j} P \frac{\partial v}{\partial x_k} + \frac{\partial^2 v^T}{\partial x_j \partial x_k} P \frac{\partial v}{\partial x_i} + \frac{\partial^2 v^T}{\partial x_i \partial x_k} P \frac{\partial v}{\partial x_j} + v^T P \frac{\partial^3 v}{\partial x_i \partial x_j \partial x_k} \]

- Approximation of function intervals by Taylor expansion
  
  - Reduce the overestimation due to interval dependency
EXAMPLE RESECTION
EXAMPLE RESECTION
EXAMPLE RESECTION – CLUSTER EFFECT

mid points of investigated intervals
CONCLUSION

- **Interval-based global optimisation** using **branch & bound - strategy** is a meaningful tool for problems having multimodal cost functions
- The correct solution of non-linear problems is found

**Pros:**
- The global optimisation yield the correct solution even if the classic non-linear adjustment (e.g. GMM) fails or leads to a local optimum only
- No requirement of adequate initial values
- Guarantee that the global optimum has been found if one exists

**Cons:**
- High computational effort
- The knowledge of high-order derivatives is useful but also costly
- Interval dependency leads to expanded interval bounds
- The computation time rapidly increases with the number of unknowns
Thanks for attention!

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