Universität Rostock Traditio et Innovatio

Chair of Mechatronics

Transformation of Dynamic Systems Into a Cooperative Form to Exploit Advantages in Interval-based Controller Design

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Contents

Why cooperativity?

To simplify

- computation of guaranteed state enclosures
- design of interval observers
- forecasting worst-case bounds for selected system outputs in predictive control
- identification of unknown parameters
- ...

Avoiding the use of general-purpose, set-valued solvers

Overestimation due to the wrapping effect may lead to (interval) bounds that are much wider than the actually reachable sets of states.

Consider the autonomous system

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)\right) \ , \quad \mathbf{x} \in \mathbb{R}^n$$

Criterion for cooperativity

Jacobian matrix

$$\mathbf{J} = \frac{\partial \mathbf{f}\left(\mathbf{x}\right)}{\partial \mathbf{x}}$$

with all off-diagonal elements $J_{i,j}$, $i, j \in \{1, \ldots, n\}$, $i \neq j$ strictly non-negative according to

$$J_{i,j} \geq 0 \ , \quad i, j \in \{1, \ldots, n\} \ , \quad i \neq j$$

Consider the autonomous system

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)\right) \ , \quad \mathbf{x} \in \mathbb{R}^n$$

Positivity of the system

Guarantee that state trajectories $\mathbf{x}(t)$ starting in the positive orthant

$$\mathbb{R}^n_+ = \{\mathbf{x} \in \mathbb{R}^n \ | \ x_i \geq 0 \quad \forall i \in \{1, \ldots, n\}\}$$

stay in this positive orthant for all $t \geq 0$ because $\dot{x}_i(t) = f_i\left(x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots x_n\right) \geq 0$ holds for all components $i \in \{1, \ldots, n\}$ of the state vector as soon as the state x_i reaches the value $x_i = 0$

Interval representation of domain of reachable states

$$[\mathbf{x}] = [\mathbf{x}](t) = \begin{bmatrix} [\underline{x}_1(t)\ ;\ \overline{x}_1(t)] \\ \vdots \\ [\underline{x}_n(t)\ ;\ \overline{x}_n(t)] \end{bmatrix}$$

with the initial states

$$[\mathbf{x}_0] = [\mathbf{x}](0) = \begin{bmatrix} [\underline{x}_1(0)\ ;\ \overline{x}_1(0)] \\ \vdots \\ [\underline{x}_n(0)\ ;\ \overline{x}_n(0)] \end{bmatrix}$$

and the vector components $[x_i] = [\underline{x}_i;\ \ \overline{x}_i]$, $i \in \{1, \ldots, n\}$,
where $\inf([x_i]) = \underline{x}_i$ is the infimum
$\sup([x_i]) = \overline{x}_i$ is the supremum

Cooperative System Models Derived From First-Principle

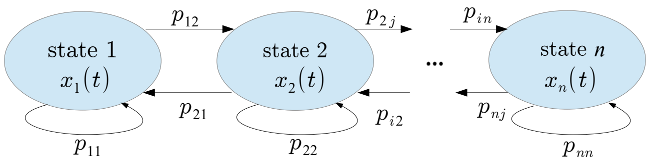

Figure: Graphical representation of a dynamic system.

Derivation of the ODEs

$$\dot{x}_i = -\sum_{j=1}^{n} p_{ij} x_i + \sum_{j=1, i\neq j}^{n} p_{ji} x_j$$

with $p_{ii} \in \mathbb{R}$, $p_{ij} \geq 0$ and $p_{ji} \geq 0$, $i \neq j$

Cooperative System Models Derived From First-Principle

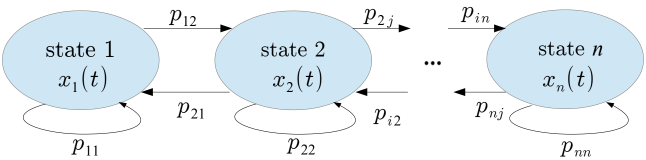

Figure: Graphical representation of a dynamic system.

State-space representation

$$\dot{\mathbf{x}} = \begin{bmatrix} -\sum_{j=1}^{n} p_{1j} & p_{21} & \cdots & p_{n1} \\ p_{12} & -\sum_{j=1}^{n} p_{2j} & \cdots & p_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1n} & p_{2n} & \cdots & -\sum_{j=1}^{n} p_{nj} \end{bmatrix} \mathbf{x} \ ,$$

Reformulation into a quasi-linear state-space representation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

For linear systems the state equations are equivalent to

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}$$

and for non-linear formulations a quasi-linear form (by factoring out selected state variables) is obtained

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}) \cdot \mathbf{x} + \mathbf{B}(\mathbf{x}) \cdot \mathbf{u}$$

Cooperativity

Here, $\mathbf{A}$ or $\mathbf{A}(\mathbf{x})$ is supposed to be Metzler and Hurwitz for asymptotically stable systems

Preparations for the transformation

$\mathbf{x} = \mathbf{x}_\mathrm{s} = \mathbf{0}$ desired operating state

$\mathbf{u} = \mathbf{u}_\mathrm{s} = \mathbf{0}$ without loss of generality for the steady-state input

with the feedback controller according to

$$\mathbf{u} = -\mathbf{K}\mathbf{x} \quad \text{or} \quad \mathbf{u} = -\mathbf{K}(\mathbf{x}) \cdot \mathbf{x}$$

leading to

$$\dot{\mathbf{x}} = (\mathbf{A}(\mathbf{x}) - \mathbf{B}(\mathbf{x}) \cdot \mathbf{K}(\mathbf{x})) \cdot \mathbf{x} = \mathbf{A}_\mathrm{C}(\mathbf{x}) \cdot \mathbf{x}$$

General transformation

$$\mathbf{z}(t) = \mathbf{\Theta}^{-1}\mathbf{x}(t) \quad \text{with} \quad \dot{\mathbf{z}}(t) = \mathbf{N} \cdot \mathbf{z}(t)$$

For general applications without diagonally dominant system matrices, the transformation consists of

$$\tilde{\mathbf{z}}(t) = \tilde{\mathbf{T}}^{-1}\mathbf{x}(t)$$

to get a diagonally dominant system matrix and

$$\mathbf{z}(t) = \mathbf{T}^{-1}\tilde{\mathbf{z}}(t) = \mathbf{T}^{-1} \cdot \tilde{\mathbf{T}}^{-1}\mathbf{x}(t) = (\tilde{\mathbf{T}} \cdot \mathbf{T})^{-1}\mathbf{x}(t)$$

to ensure a Metzler structure, resulting in the overall transformation matrix $\mathbf{\Theta} = \tilde{\mathbf{T}} \cdot \mathbf{T}$

Structure of the transformation matrix

$\boldsymbol{\Theta}$ may be a **time-invariant** or **time-varying** matrix according to the following distinction

Systems With Purely Real Eigenvalues

Preliminary

$$\mathbf{Z}_\mathrm{a} - \mathbf{\Delta} \leq \mathbf{Z} := \mathbf{A}_\mathrm{C} \leq \mathbf{Z}_\mathrm{a} + \mathbf{\Delta}$$

with $\mathbf{\Delta}$, which consists of the (symmetric) worst-case bounds of all entries in $[\mathbf{A}]_\mathrm{C}$ and $\mathbf{Z}_\mathrm{a} = \mathbf{Z}_\mathrm{a}^T$ as a symmetric midpoint matrix and

$$\mathbf{R} = \mu \mathbf{E}_n - \mathbf{\Gamma}$$

as Metzler matrix, which has the same eigenvalues as $\mathbf{Z}_\mathrm{a}$

with

$\mu \in \mathbb{R}$	constant
$\mathbf{\Gamma} \in \mathbb{R}^{n \times n}$	diagonal matrix
$\mathbf{E}_n \in \mathbb{R}^{n \times n}$	matrix with all elements equal to 1 and
$\mathbf{\Gamma} = \rho \mathbf{I}_n$	with $\rho > \mu$ and the identity matrix $\mathbf{I}$ of order n

Assumption

If

$$\mathrm{eig}(\mathbf{R}) = \mathrm{eig}(\mathbf{Z}_\mathrm{a})$$

there exists an orthogonal matrix $\mathbf{S} \in \mathbb{R}^{n\times n}$ such that $\mathbf{S}^T\mathbf{Z}\mathbf{S}$ or $\mathbf{\Theta}^T\mathbf{Z}\mathbf{\Theta}$, respectively, is Metzler provided that

$$\mu > n||\mathbf{\Delta}||_\mathrm{max} \ ,$$

where $||\mathbf{\Delta}||_\mathrm{max}$ denotes the maximum absolute value of $\mathbf{\Delta}$

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$$\mu > n||\mathbf{\Delta}||_\mathrm{max} \ ,$$

where $||\mathbf{\Delta}||_\mathrm{max}$ denotes the maximum absolute value of $\mathbf{\Delta}$

Aim

Computationally feasible optimization problem formulated with LMI constraints to find a suitable transformation matrix $\mathbf{S}$

Choosing a diagonal matrix $\mathbf{Z}_a$

If the system is

1. diagonally dominant: $\mathbf{Z}_{\mathrm{a}}$ represents the diagonal entries of the original system matrix
2. not diagonally dominant: $\mathbf{Z}_{\mathrm{a}}$ is a diagonal matrix with the asymptotically stable, real eigenvalues of $\mathrm{mid}\{[\mathbf{A}]_{\mathrm{C}}\}$, which is determined by a new matrix

$$\hat{\mathbf{A}}_{\mathrm{C}} = \tilde{\mathbf{T}}^{-1}\mathbf{A}_{\mathrm{C}}\tilde{\mathbf{T}}$$

such that the element-wise defined interval midpoint matrix $\mathrm{mid}\{[\mathbf{A}]_{\mathrm{C}}\}$ is transformed into a diagonal structure (except for numerical round-off errors). If $\mathrm{mid}\{[\mathbf{A}]_{\mathrm{C}}\}$ possesses n linearly independent real-valued eigenvectors, their floating-point approximation is used to define the matrix $\tilde{\mathbf{T}}$.

Choosing $\boldsymbol{\Delta}$

$$\delta = \max\left(|[\mathbf{A}]_{\mathrm{C}} - \mathbf{Z}_{\mathrm{a}}|\right) \quad \text{or} \quad \delta = \max\left(|[\hat{\mathbf{A}}]_{\mathrm{C}} - \mathbf{Z}_{\mathrm{a}}|\right)$$

with $\boldsymbol{\Delta} = \delta \cdot \mathbf{E}_n$

Further specifications

$$\mu^{\star} = n||\boldsymbol{\Delta}||_{\max}$$

marks the lower bound for μ and

$$\mathbf{R} = \mathbf{S}^T \mathbf{Z}_{\mathrm{a}} \mathbf{S} \quad \text{and} \quad \mathbf{S}^T \mathbf{S} = \mathbf{I}$$

need to be satisfied

Reformulation Into an Optimization Problem

Orthogonality of S

$$-\mathbf{R} + \mathbf{S}^T \mathbf{Z}_{\mathrm{a}} \mathbf{S} \succ \mathbf{0} \quad \text{and} \quad \mathbf{I} - \mathbf{S}^T \mathbf{S} \succ \mathbf{0}$$

is converted by application of the Schur complement formula according to

$$\begin{bmatrix} -\mathbf{R} & \mathbf{S}^T \\ \mathbf{S} & -\mathbf{Z}_{\mathrm{a}}^{-1} \end{bmatrix} \succ \mathbf{0} \quad \text{and} \quad \begin{bmatrix} \mathbf{I} & \mathbf{S}^T \\ \mathbf{S} & \mathbf{I} \end{bmatrix} \succ \mathbf{0}$$

Reformulation Into an Optimization Problem

Known specifications to other variables

$$\mathbf{R} = \bar{\mu}\mathbf{E}_n - \mathbf{\Gamma} \ , \quad \bar{\mu} > \mu$$

where the LMI constraints

$$\mathbf{\Gamma} \succ \mathbf{0} \quad \text{and} \quad \mathbf{R}^T\mathbf{Q} + \mathbf{Q}\mathbf{R} \prec \mathbf{0}$$

with $\mathbf{Q} \succ \mathbf{0}$ (Hurwitz stability of $\mathbf{R}$)

Overall cost function

$$J = \mathrm{tr}(\mathbf{\Gamma}) + \mathrm{tr}(\mathbf{Z}_\mathrm{a}\mathbf{S} - \breve{\mathbf{S}}\mathbf{R}) - \kappa \cdot \mathrm{tr}(\breve{\mathbf{S}}^T\mathbf{S} - \mathbf{I})$$

with the problem-dependent parameter $\kappa > 0$ and the solution of the last successful evaluation of the LMI-constrained optimization task $\breve{\mathbf{S}}$

Systems With Purely Conjugate-Complex Eigenvalues

Preliminaries

- Generally only time-varying transformations possible (exception are exactly known systems)
- Uncertainty is mapped into the position of the eigenvalues

Systems With Purely Conjugate-Complex Eigenvalues

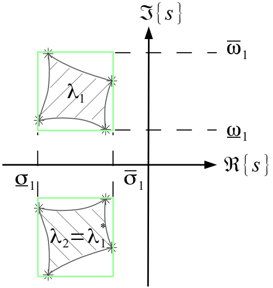

Figure: Possible positions of conjugate-complex eigenvalues.

Interval hull

$[\sigma_i] = [\underline{\sigma}_i;\ \ \overline{\sigma}_i]$ and $[\omega_i] = [\underline{\omega}_i;\ \ \overline{\omega}_i]$

Note

The presented approach is only valid for disjoint eigenvalue domains

Transformation matrix

The number of considered eigenvalues is reduced to

$$\tilde{n} = \frac{n}{2}$$

for a system with n states, because of conjugate-complex pairs

$$\tilde{\mathbf{T}} = \left[\tilde{\mathbf{T}}_1, \ldots, \tilde{\mathbf{T}}_{\tilde{n}}\right], \text{ where } \tilde{\mathbf{T}}_j = [\Re\{[\mathbf{v}_j]\}, \Im\{[\mathbf{v}_j]\}]$$

with $j \in \{1, \ldots, \tilde{n}\}$

Transformed system

Results in the real-valued Jordan canonical form

$$\tilde{\mathbf{A}} = \mathrm{blkdiag}\left(\tilde{\mathbf{A}}_1, \ldots, \tilde{\mathbf{A}}_{\tilde{n}}\right) \text{ with } \tilde{\mathbf{A}}_j = \begin{bmatrix} [\sigma_j] & [\omega_j] \\ -[\omega_j] & [\sigma_j] \end{bmatrix}.$$

The time-variant transformation is done by

$$\mathbf{z} = \mathbf{T}^{-1}(t) \cdot \tilde{\mathbf{z}} \quad \text{with}$$

$$\mathbf{T}^{-1}(t) = \mathrm{blkdiag}\left(\mathbf{T}_1^{-1}(t), \ldots, \mathbf{T}_{\tilde{n}}^{-1}(t)\right) = \mathbf{T}^T(t)$$

and the orthogonal blocks

$$\mathbf{T}_j = \begin{bmatrix} \cos([\omega_j]t) & \sin([\omega_j]t) \\ -\sin([\omega_j]t) & \cos([\omega_j]t) \end{bmatrix}$$

for $j \in \{1, \ldots, \tilde{n}\}$

Resulting state-space representation of the system

$$\dot{\mathbf{z}} = \dot{\mathbf{T}}^T(t) \cdot \tilde{\mathbf{z}} + \mathbf{T}^T(t) \cdot \dot{\tilde{\mathbf{z}}}$$
$$= \left[\left[\frac{\mathrm{d}\mathbf{T}^T(t)}{\mathrm{d}t} + \mathbf{T}^T(t)\tilde{\mathbf{A}} \right] \mathbf{T}(t) \right] \mathbf{z} = \mathbf{N} \cdot \mathbf{z}$$

Regarding the structure of N

Symbolic simplifications in terms of the exact values ω_j^*

$$\mathbf{N} = \mathrm{blkdiag}\left(\sigma_1 \mathbf{I}, \ldots, \sigma_{\tilde{n}} \mathbf{I}\right) \ , \quad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Resulting state-space representation of the system

$$\dot{\mathbf{z}} = \dot{\mathbf{T}}^T(t) \cdot \tilde{\mathbf{z}} + \mathbf{T}^T(t) \cdot \dot{\tilde{\mathbf{z}}}$$
$$= \left[\left[\frac{\mathrm{d}\mathbf{T}^T(t)}{\mathrm{d}t} + \mathbf{T}^T(t)\tilde{\mathbf{A}} \right] \mathbf{T}(t) \right] \mathbf{z} = \mathbf{N} \cdot \mathbf{z}$$

Asymptotic stability

Since $\mathbf{N}$ depends on the system's eigenvalues, Hurwitz stability is guaranteed for $\overline{\sigma}_j < 0$. Extrema of the conjugate-complex eigenvalues are obtained by building the hull over their real and imaginary parts

$$[\sigma_j] = [\min(\sigma_j); \quad \max(\sigma_j)] \ ,$$
$$[\omega_j] = [\min(\omega_j); \quad \max(\omega_j)] \ .$$

System With Purely Real Eigenvalues

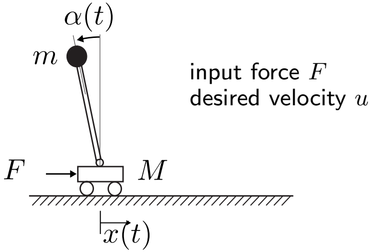

Figure: Control of an inverted pendulum on a moving carriage.

Nonlinear differential equations

$$ma^2 \cdot \ddot{\alpha} - ma \cdot \cos(\alpha) \cdot \ddot{x} - mga \cdot \sin(\alpha) = 0,$$

$$(M + m) \cdot \ddot{x} - ma \cdot \cos(\alpha) \cdot \ddot{\alpha} + ma \cdot \sin(\alpha) \cdot \dot{\alpha} = F$$

Quasi-linear state-space representation

Introducing an underlying velocity control with u as the desired carriage velocity

$$T_1 \cdot \ddot{x} + \dot{x} = u$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{g \cdot \mathrm{si}(\alpha)}{a} & 0 & 0 & -\frac{\cos(\alpha)}{T_1 a} \\ 0 & 0 & 0 & -\frac{1}{T_1} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \frac{\cos(\alpha)}{T_1 a} \\ \frac{1}{T_1} \end{bmatrix} u$$

$$y = \begin{bmatrix} -a \cdot \mathrm{si}(\alpha) & 1 & 0 & 0 \end{bmatrix} \mathbf{x} \,, \quad \mathrm{si}\,(\alpha) = \frac{\sin(\alpha)}{\alpha}$$

with the state vector $\mathbf{x} = \begin{bmatrix} \alpha & x & \dot{\alpha} & \dot{x} \end{bmatrix}^T$ and the system input u

Controlled system for $[\alpha] = [51.4°; \ \ 51.7°]$

$$\mathbf{A}_{\mathrm{C}}(\alpha) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31}(\alpha) - k_1 b_3(\alpha) & -k_2 b_3(\alpha) & -k_3 b_3(\alpha) & a_{34}(\alpha) - k_4 b_3(\alpha) \\ -k_1 b_4 & -k_2 b_4 & -k_3 b_4 & a_{44} - k_4 b_4 \end{bmatrix}$$

$$[a]_{31} = [-1.25; \ \ -1.23] \cdot 10^3 \qquad [a]_{32} = [12.81; \ \ 12.91]$$
$$[a]_{33} = [-2.55; \ \ -2.52] \cdot 10^2 \qquad [a]_{34} = [1.07; \ \ 1.09] \cdot 10^2$$
$$[a]_{41} = [-4.13; \ \ -4.12] \cdot 10^2 \qquad [a]_{42} = [4.13; \ \ 4.14]$$
$$[a]_{43} = [-81.49; \ \ -81.48] \qquad [a]_{44} = [34.70; \ \ 34.71]$$

Transformation matrix

$$\boldsymbol{\Theta} = \mathbf{VS} = \begin{bmatrix} -0.016 & -0.170 & -0.016 & 0.155 \\ 0.998 & 0.694 & -0.104 & -0.462 \\ 0.056 & 0.309 & 1.095 & -0.012 \\ 0.283 & -0.253 & -0.504 & 0.740 \end{bmatrix}$$

Transformed system

$$\mathbf{N} = \tilde{\mathbf{A}}_{\mathrm{C}} \in 1 \cdot 10^2 \begin{bmatrix} [a]_{11} & [a]_{12} & [a]_{13} & [a]_{14} \\ [a]_{21} & [a]_{22} & [a]_{23} & [a]_{24} \\ [a]_{31} & [a]_{32} & [a]_{33} & [a]_{34} \\ [a]_{41} & [a]_{42} & [a]_{43} & [a]_{44} \end{bmatrix} \quad \text{with}$$

$$[a]_{11} = [-2.124;\ -2.100] \qquad [a]_{12} = [\ 0.143;\ 0.175]$$
$$[a]_{21} = [\ 0.152;\ 0.166] \qquad [a]_{22} = [-0.057;\ -0.038]$$
$$[a]_{31} = [\ 0.064;\ 0.067] \qquad [a]_{32} = [\ 0.001;\ 0.005]$$
$$[a]_{41} = [\ 0.026;\ 0.038] \qquad [a]_{42} = [\ 0.001;\ 0.018]$$

$$[a]_{13} = [\ 0.061;\ 0.070] \qquad [a]_{14} = [\ 0.016;\ 0.047]$$
$$[a]_{23} = [\ 0.000;\ 0.006] \qquad [a]_{24} = [\ 0.000;\ 0.018]$$
$$[a]_{33} = [-0.010;\ -0.008] \qquad [a]_{34} = [\ 0.000;\ 0.004]$$
$$[a]_{43} = [\ 0.000;\ 0.005] \qquad [a]_{44} = [-0.024;\ -0.008]$$

System With Purely Conjugate-Complex Eigenvalues

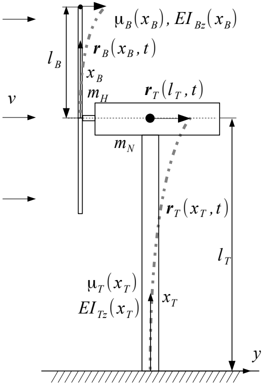

Figure: Mechanical model of the wind turbine with an elastic tower.

Ordinary differential equations

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{\mathrm{TB}}^{-1}\mathbf{K}_{\mathrm{TB}} & -\mathbf{M}_{\mathrm{TB}}^{-1}(\mathbf{D}_{\mathrm{TB}} + \mathbf{h}_{\mathrm{TB}}\mathbf{k}_{\mathrm{T}}^{T}) \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}$$

with uncertainties in $\mathbf{M}_{TB}$, $\mathbf{D}_{TB}$ and $\mathbf{K}_{TB}$ due to their dependency on ω_R, k_{dT} and k_{dB}

Parameter domains

$$[\omega_{\mathrm{R}}] = [0.7; \quad 1.4]\ \mathrm{s}^{-1}$$

$$[k_{\mathrm{dT}}] = [2.5; \quad 3.5] \cdot 10^{-2}\ \mathrm{N} \cdot \mathrm{s/m}$$

$$[k_{\mathrm{dB}}] = [0.5; \quad 1.5] \cdot 10^{-2}\ \mathrm{N} \cdot \mathrm{s/m}$$

Transformation

$$[\mathbf{N}] = \mathrm{diag}\left([\sigma_1], [\sigma_1], [\sigma_3], [\sigma_3]\right)$$

with

$$[\sigma_1] = [-0.105; -0.016]$$

$$[\omega_1] = [3.893; 5.310]$$

$$[\sigma_3] = [-0.068; -0.040]$$

$$[\omega_3] = [1.875; 1.908]$$

Transformation of the initial conditions

$$\tilde{\mathbf{T}} = [v_1^{\Re}, \quad v_1^{\Im}, \quad v_3^{\Re}, \quad v_3^{\Im}]$$

$$[\mathbf{x}](0) = \begin{bmatrix} [1.25; \quad 1.5] \\ [0.25; \quad 0.5] \\ [0] \\ [0] \end{bmatrix} \quad \xrightarrow{\tilde{\mathbf{T}}} \quad [\mathbf{z}](0) = \begin{bmatrix} [-0.761; \quad 0.335] \\ [-9.792; \quad 1.608] \\ [-0.297; \quad 0.619] \\ [\ \ 2.609; \quad 5.474] \end{bmatrix}$$

System with real and complex eigenvalues

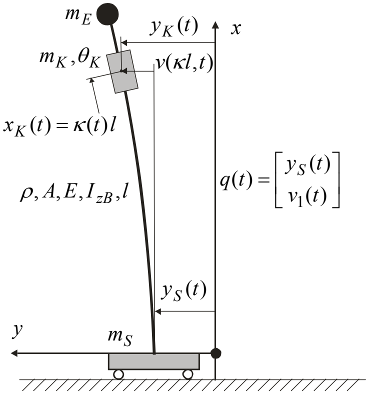

Figure: Mechanical model of the stacker crane.

Ordinary differential equations

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{h} \cdot [F_{SM} - F_{SR}(\dot{y}_S)]$$

with uncertainties in $\mathbf{M}$ and $\mathbf{K}$ due to their dependency on κ, a dimensionless Parameter to consider the varying vertical position of the payload x_K

$$\kappa = \frac{x_K}{l}$$

Parameter domains

$$[\kappa] = [0.45; \quad 0.54] \text{ m}$$

Transformation

$$[\mathbf{N}] = \mathrm{diag}\left([\sigma_1], [\sigma_2], [\sigma_2], [\sigma_4], [\sigma_5], [\sigma_5]\right)$$

with

$$[\sigma_1] = [-601.4; -560.1]$$

$$[\sigma_2] = [-27.4; -25.7]$$

$$[\omega_2] = [125.9; 127.5]$$

$$[\sigma_4] = [-7.5; -6.8]$$

$$[\sigma_5] = [-20.2; -17.5]$$

$$[\omega_5] = [19.8; 22.5]$$

Conclusions

- Using advantages of cooperative systems
  - avoiding the wrapping effect
  - simulating lower and upper bounds individually
  - reflecting the characteristics of exactly known systems
- purely real eigenvalues: LMI approach to satisfy given requirements
- complex eigenvalues: symbolically proven solution

Conclusions

- Using advantages of cooperative systems
  - avoiding the wrapping effect
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  - reflecting the characteristics of exactly known systems
- purely real eigenvalues: LMI approach to satisfy given requirements
- complex eigenvalues: symbolically proven solution

Outlook

- Optimization of the line-search procedure for the parameter μ
- Performance improvement for higher-dimensional applications
- Extensions to systems with real and conjugate-complex eigenvalues as well as multiple eigenvalues in a joint approach combining both presented procedures

Thank you for your attention!