Multi-robots pose domain characterization using interval methods

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Multi-robots Cooperative pose domain characterization

Problem statement

- Uncertain measurements
  - Accurate altitude
  - Accurate roll & pitch
  - Rough heading

- 2D-3D corresponding points camera measurements
- Inter-distance measurements $d_{k,j}$
- Distance to a given base station $d_k$

- Robots communicate with each other
  - Exchange measurements

- Determine the poses $k \in \{1, 2\}$
  $$\mathbf{r}_k = (x_k, y_k, z_k, \phi_k, \theta_k, \psi_k)$$
Outline

Multi-robot cooperative localization

- Bounded-error measurements
- Constraints network
- Solving strategy
- Simulation & experimental results

Conclusion and outlook
Multi-robots Cooperative pose domain characterization

**Bounded-error measurements**

- 2D-3D corresponding points measurements
  \[ \bar{X}_{i,k} \in [\bar{X}_{i,k}] \quad wX_i \in [wX_i] \]

- Inter-distance measurements
  \[ d_{k,j} \in [d_{k,j}] \quad j \in \mathcal{N}(k) \]

- Distance to the base station
  \[ d_k \in [d_k] \]

- Proprioceptive data
  \[
  \begin{bmatrix}
  \phi_{\text{meas.}} & \pm \epsilon_{\phi} \\
  \theta_{\text{meas.}} & \pm \epsilon_{\theta} \\
  z_{\text{meas.}} & \pm \epsilon_{z}
  \end{bmatrix}
  \]
Multi-robots Cooperative pose domain characterization

Set inversion with contractors

- Set inversion
  - Given $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$
    $$\mathbf{X} = \{ \mathbf{x} \in \mathbb{R}^n | f(\mathbf{x}) \in \mathbf{Y} \} = \mathbf{f}^{-1}(\mathbf{Y})$$
  - Inclusion function $\forall [\mathbf{x}] \in \mathbb{IR}^n, f([\mathbf{x}]) \subset [f([\mathbf{x}])]$
  - SIVIA : branch and bound algorithm
    - If $[f']$ is convergent SIVIA output $\mathbf{X}^- \subset \mathbf{X} \subset \mathbf{X}^+$

- Contractors
  - $\forall [\mathbf{x}] \in \mathbb{IR}^n, C([\mathbf{x}]) \subseteq [\mathbf{x}]$ contraction
  - $(\mathbf{x} \in [\mathbf{x}], C(\mathbf{x}) = \mathbf{x}) \Rightarrow \mathbf{x} \in C([\mathbf{x}])$ consistency
  - $C(\mathbf{x}) = \emptyset \Leftrightarrow (\exists \varepsilon > 0, \forall [\mathbf{x}] \subseteq B(\mathbf{x}, \varepsilon), C([\mathbf{x}]) = \emptyset)$ continuity

- Result:
  Outer approximation for set of all poses compatible with measurements $\rightarrow$ SIVIA+Contractors
Multi-robots Cooperative pose domain characterization

Constraints network

- **Camera constraints**

\[
C_i : \begin{cases} 
(cX_i, cY_i, cZ_i) = c^T r^r T_w(r)^w X_i \\
     c x_i = \frac{cX_i}{cZ_i}, c y_i = \frac{cY_i}{cZ_i}, \\
     c x_i \in [c x_i], c y_i \in [c y_i], c Z_i > 0 
\end{cases}
\]

- **Inter-distances constraints**

\[
d_{k,j} = \| p_k - p_j \|_2
\]

- **Base distance constraints**

\[
d_k = \| p_k - b \|_2
\]
Distributed pose domain characterization

Constraints satisfaction problems

- **Initial pose estimation**
  - Altitude & IMU angles set in initial domain \([r]\)
  - Image measurement using CSP

\[
\mathcal{H}_{\text{img}} : \left( \begin{array}{c}
  r \in [r], \\
  \{C_i, \ i \in 1...m\}
\end{array} \right) \rightarrow \mathcal{S}_{r_k}^+ \quad \text{Outer subpaving for robot } R_k
\]

- **Refine pose** using CSP for distance / inter-distances constraints

\[
\mathcal{H}_k : \begin{cases}
  p_k \in \text{proj}_P(\mathcal{S}_{r_k}^+), \\
  p_j \in [p_j], \ j \in \mathcal{N}(k) \\
  d_{k,j} \in [d_{k,j}], \ j \in \mathcal{N}(k) \\
  d_{k,j} = \|p_k - p_j\|_2, \ j \in \mathcal{N}(k)
\end{cases}
\]
Distributed pose domain characterization

Computation strategy for each robot

- Image only computation
- Transmit and Receive neighbor robots bounding boxes
- Refine pose domain using Distances only
- Output Fixed point

\[
[p_k] = \bigoplus \text{proj}_p S_{r_k}^+
\]

**H**

\[
H_1 \quad H_2 \quad \ldots \quad H_k
\]

\[
S_{r_1}^+, \quad S_{r_2}^+, \quad \ldots \quad S_{r_k}^+
\]
Results for two robots

Simulation & experimental results

Cooperative localisation with two drones

We now consider 2 robots $R1$ and $R2$. The robots exchange their measurements. An additional measurement is given by the inter-distance $d_{1,2}$.
Computing a point estimate

Computing a bounding box

CoB
Results for two robots

Simulation results & Comparison with LM

Mean horizontal error full visibility

Mean horizontal error reduced visibility
Distributed pose domain characterization

Experimental results: subpaving in full visibility case
Distributed pose domain characterization

Experimental results: subpaving in full visibility case
Distributed pose domain characterization

Experimental results: subpaving in reduced visibility case
Distributed pose domain characterization

**Experimental results:** One robot result in the case of 4 robots

**Horizontal position domain width (m)**

**Average horizontal position error width (m)**
Conclusion and outlook

- CoB is a good point estimate
- CoB is good initial guess for LM and EKF
- More precise positioning with growing number of robots
- Localization possible in case of complete reduced visibility for some robots due to position exchanges
Thanks for listening!

Question?