Multi-robots pose domain characterization using interval methods

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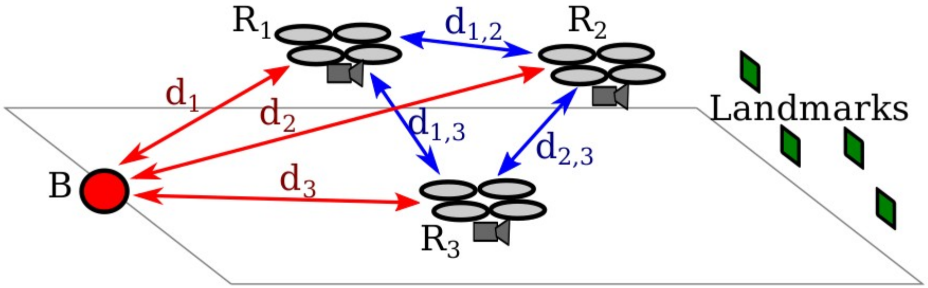
SWIM 2018 : 25th – 27th July, Rostock

Multi-robots Cooperative pose domain characterization

Problem statement



- Uncertain measurements
  - Accurate altitude
  - Accurate roll & pitch
  - Rough heading

  - 2D-3D corresponding points camera measurements
  - Inter-distance measurements $d_{k,j}$
  - Distance to a given base station d_k

- Robots communicate with each other
  - Exchange measurements

- Determine the poses $k \in \{1, 2\}$

$$\mathbf{r}_k = (x_k, y_k, z_k, \phi_k, \theta_k, \psi_k)$$

Outline

Multi-robot cooperative localization

- Bounded-error measurements
- Constraints network
- Solving strategy
- Simulation & experimental results

Conclusion and outlook

Multi-robots Cooperative pose domain characterization

Bounded-error measurements

- 2D-3D corresponding points measurements

$$\bar{\mathbf{x}}_{i,k} \in [\bar{\mathbf{x}}_{i,k}] \qquad {}^{w}\mathbf{X}_i \in [{}^{w}\mathbf{X}_i]$$

- Inter-distance measurements

$$d_{k,j} \in [\, d_{k,j} \,] \qquad j \in \mathcal{N}(k)$$

- Distance to the base station $d_k \in [d_k]$

- Proprioceptive data

$$[\phi^{\text{meas.}} \pm \epsilon_\phi]$$
$$[\theta^{\text{meas.}} \pm \epsilon_\theta]$$
$$[z^{\text{meas.}} \pm \epsilon_z]$$

Multi-robots Cooperative pose domain characterization

Set inversion with contractors

- Set inversion
  - Given $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^n | \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y})$$

  - Inclusion function $\forall [\mathbf{x}] \in \mathbb{IR}^n, \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{x}])$
  - SIVIA : branch and bound algorithm
    - If $[\mathbf{f}]$ is convergent SIVIA output $\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+$

- Contractors
  - $\forall [\mathbf{x}] \in \mathbb{IR}^n, C([\mathbf{x}]) \subseteq [\mathbf{x}]$ contraction
  - $(\mathbf{x} \in [\mathbf{x}], C(\mathbf{x}) = \mathbf{x}) \Rightarrow \mathbf{x} \in C([\mathbf{x}])$ consistency
  - $C(\mathbf{x}) = \emptyset \Leftrightarrow (\exists \varepsilon > 0, \forall [\mathbf{x}] \subseteq B(\mathbf{x}, \varepsilon), C([\mathbf{x}]) = \emptyset)$ continuity
- Result :
  Outer approximation for set of all poses compatible with measurements → *SIVIA+Contractors*

Multi-robots Cooperative pose domain characterization

Constraints network

- **Camera constraints**

$$C_i : \begin{cases} ({}^cX_i, {}^cY_i, {}^cZ_i) = {}^c\mathbf{T}_r \, {}^r\mathbf{T}_w(\mathbf{r}) \, {}^w\mathbf{X}_i \\ {}^cx_i = \frac{{}^cX_i}{{}^cZ_i}, {}^cy_i = \frac{{}^cY_i}{{}^cZ_i}, \\ {}^cx_i \in [{}^cx_i], \; {}^cy_i \in [{}^cy_i], {}^cZ_i > 0 \end{cases}$$

- **Inter-distances constraints**

$$d_{k,j} = \|\mathbf{p}_k - \mathbf{p}_j\|_2$$

- **Base distance constraints**

$$d_k = \|\mathbf{p}_k - \mathbf{b}\|_2$$

Distributed pose domain characterization

Constraints satisfaction problems

- **Initial pose estimation**
  - Altitude & IMU angles set in initial domain $[\mathbf{r}]$
  - Image measurement using CSP

$$\mathcal{H}_{\text{img}} : \begin{pmatrix} \mathbf{r} \in [\mathbf{r}], \\ \{C_i,\ i \in 1 \ldots m\} \end{pmatrix} \longrightarrow \mathbb{S}^{+}_{\mathbf{r}_k}$$

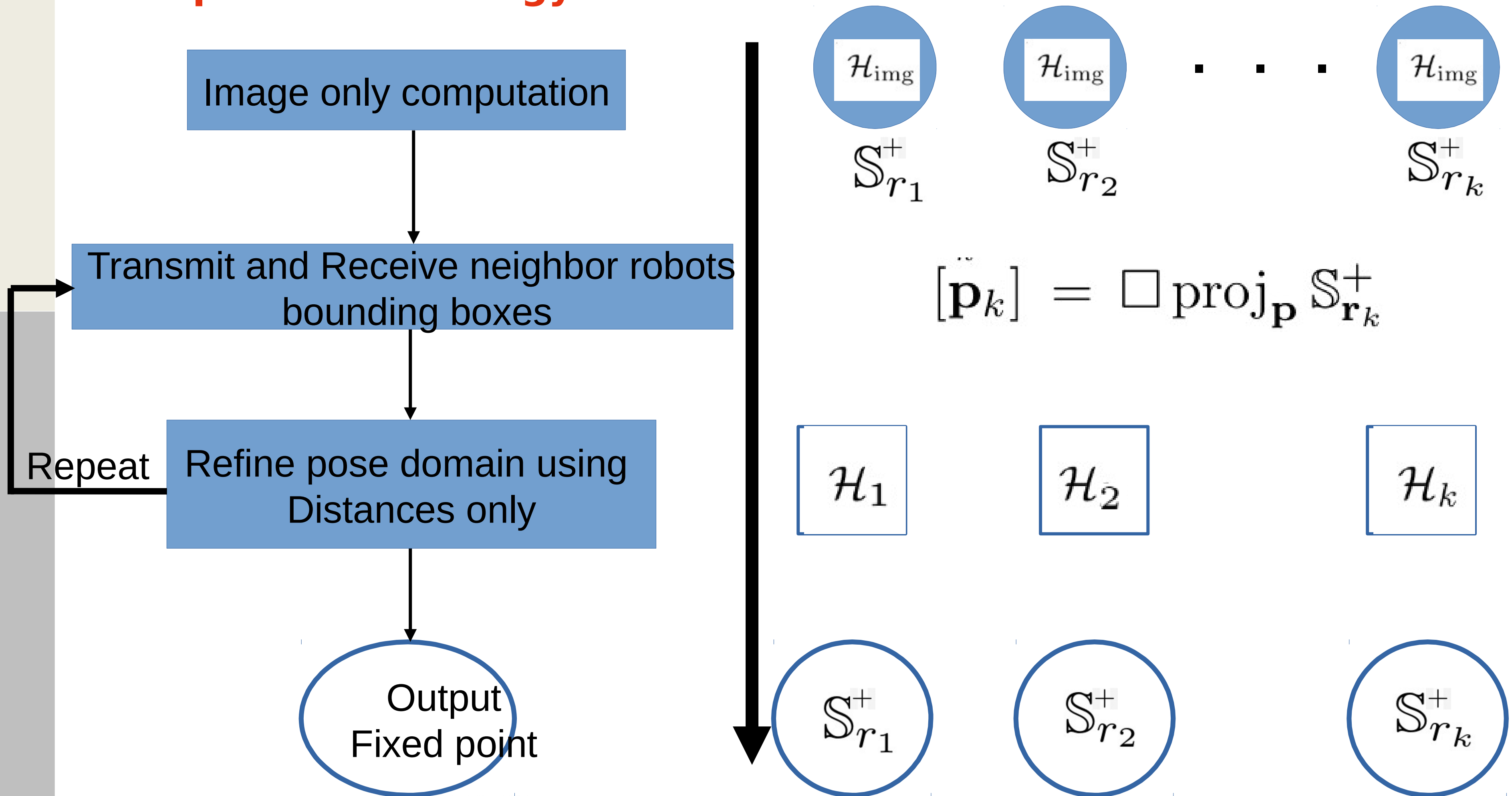
Outer subpaving for robot R_k

-
- **Refine pose** using CSP for distance / inter-distances constraints

$$\mathcal{H}_k : \begin{pmatrix} \mathbf{p}_k \in \text{proj}_{\mathbf{p}}(\mathbb{S}^{+}_{\mathbf{r}_k}), \\ \mathbf{p}_j \in [\mathbf{p}_j],\ j \in \mathcal{N}(k) \\ d_{k,j} \in [d_{k,j}],\ j \in \mathcal{N}(k) \\ d_{k,j} = \|\mathbf{p}_k - \mathbf{p}_j\|_2,\ j \in \mathcal{N}(k) \end{pmatrix}$$

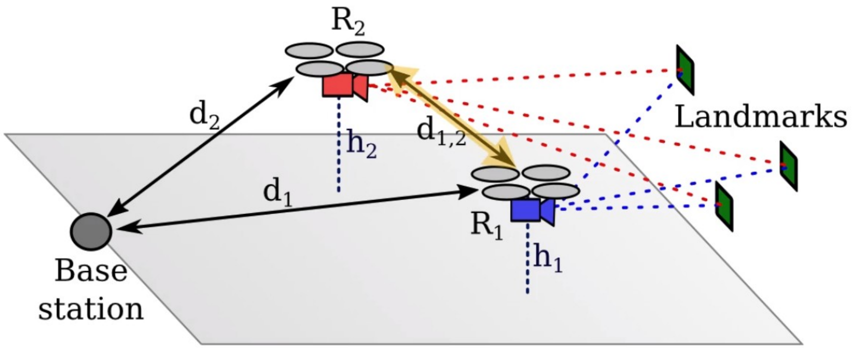
Distributed pose domain characterization

Computation strategy for each robot



Results for two robots

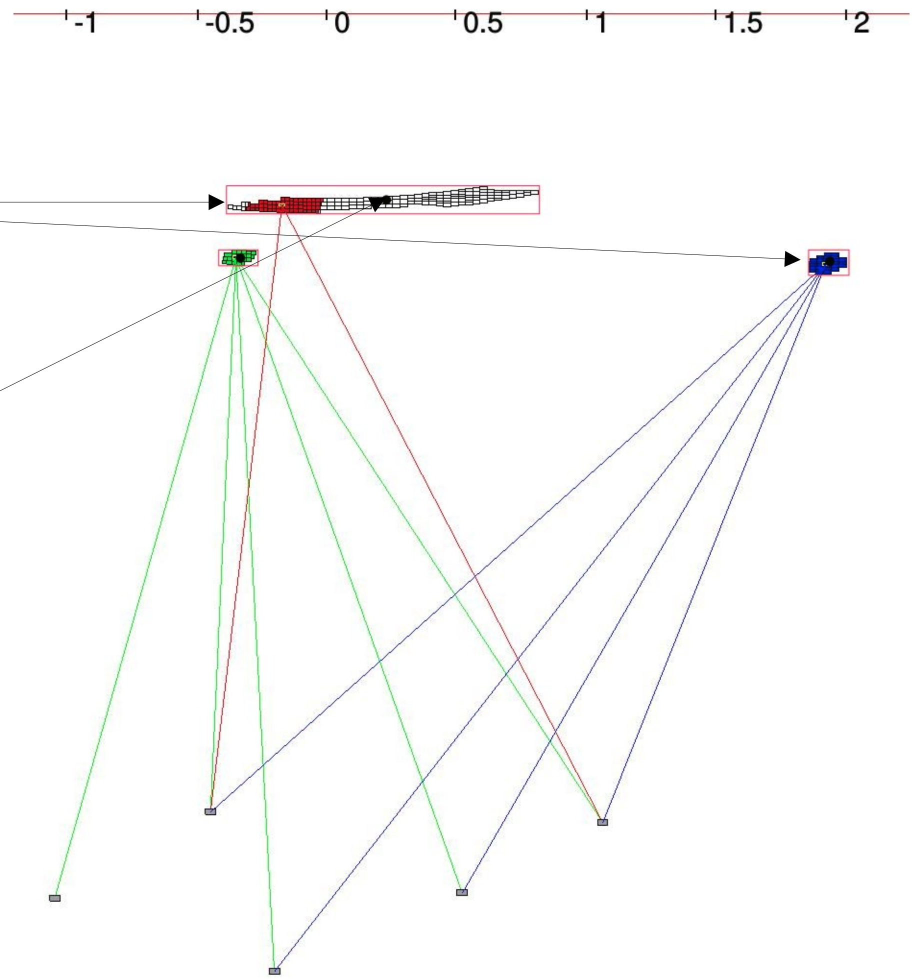
Simulation & experimental results



Cooperative localisation with two drones

We now consider 2 robots **R1** and **R2**.
The robots exchange their measurements.
An additional measurement is given by the inter-distance **$d_{1,2}$**

Computing a point estimate

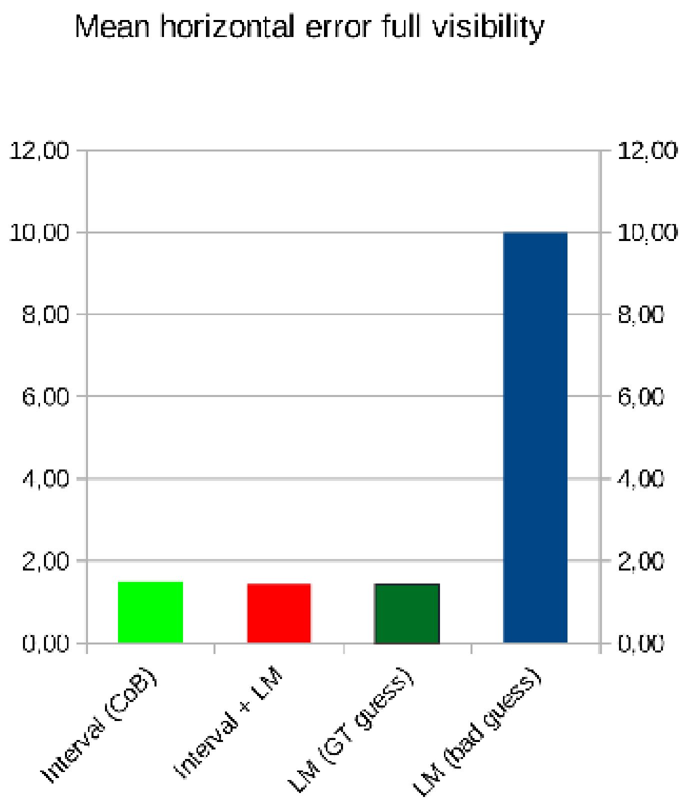


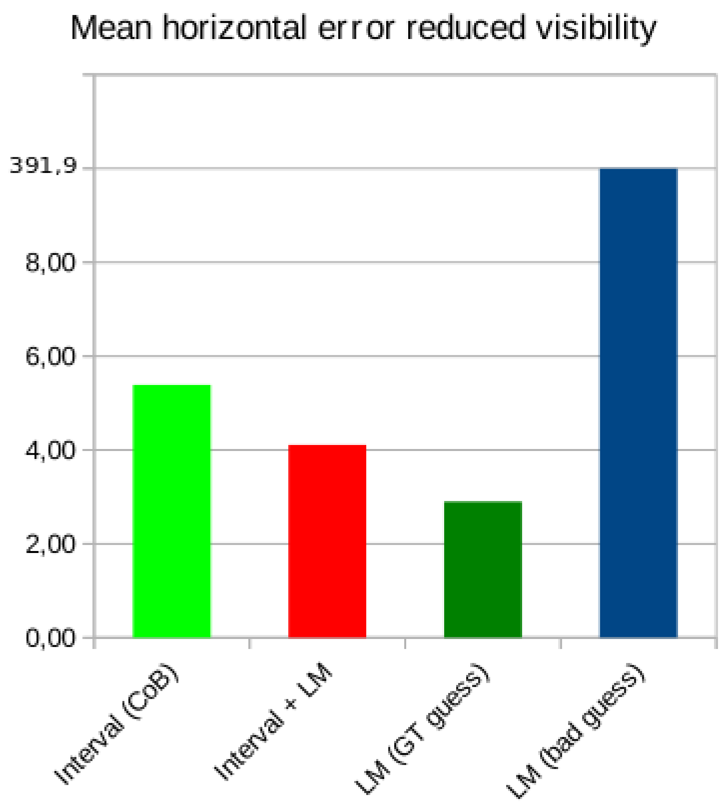
Computing a bounding box

CoB

Results for two robots

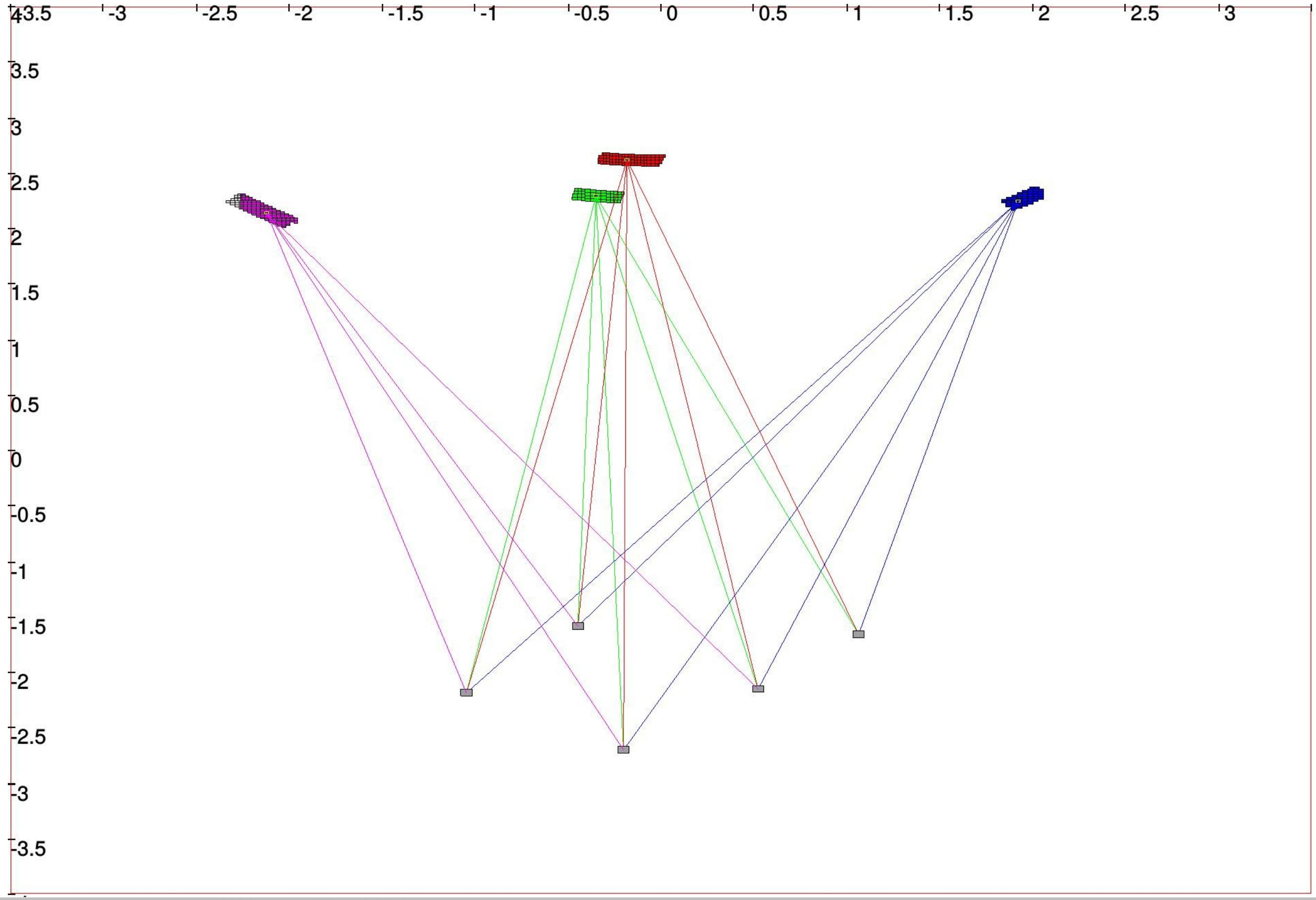
Simulation results & Comparison with LM





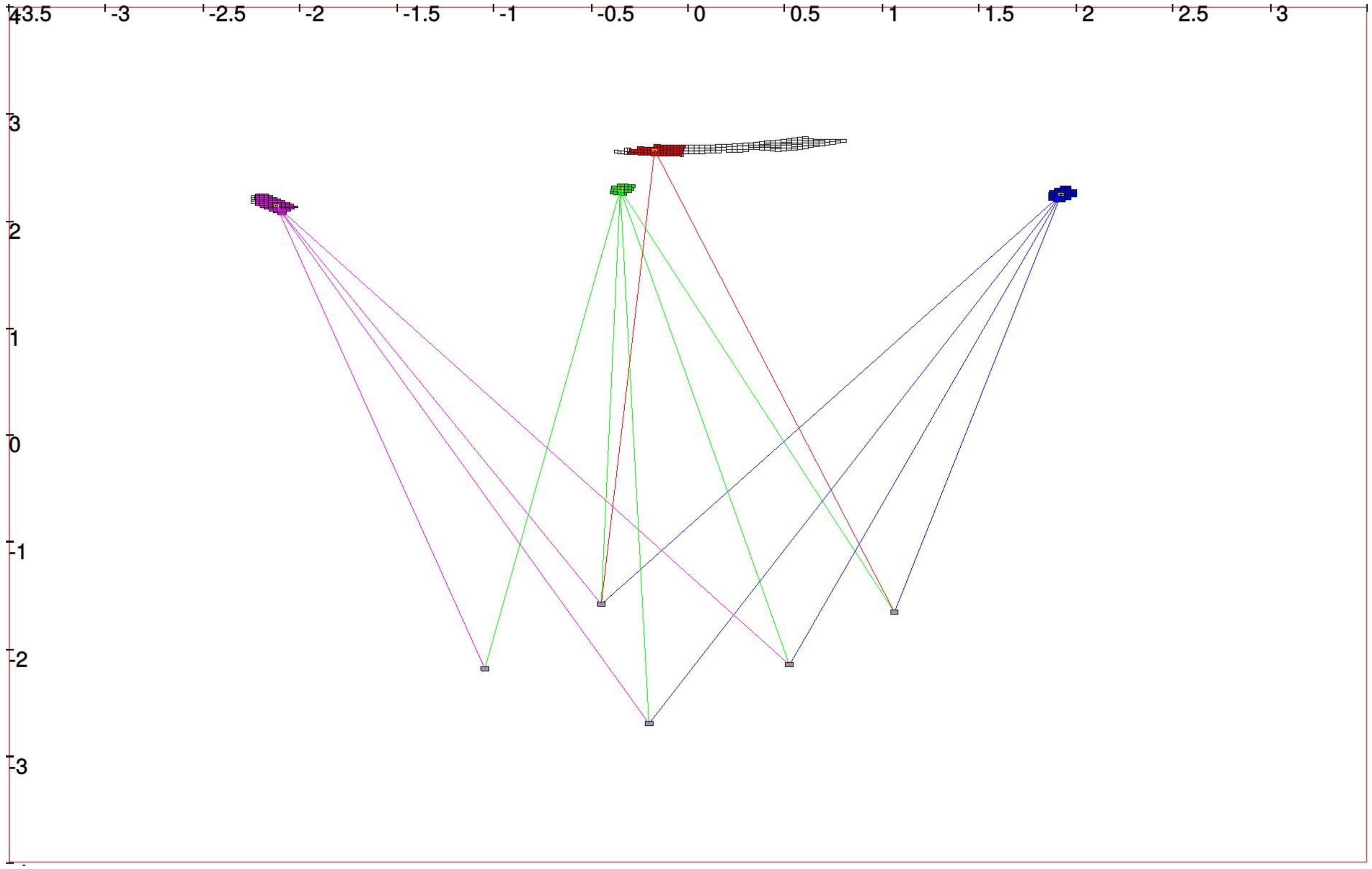
Distributed pose domain characterization

Experimental results : subpaving in full visibility case



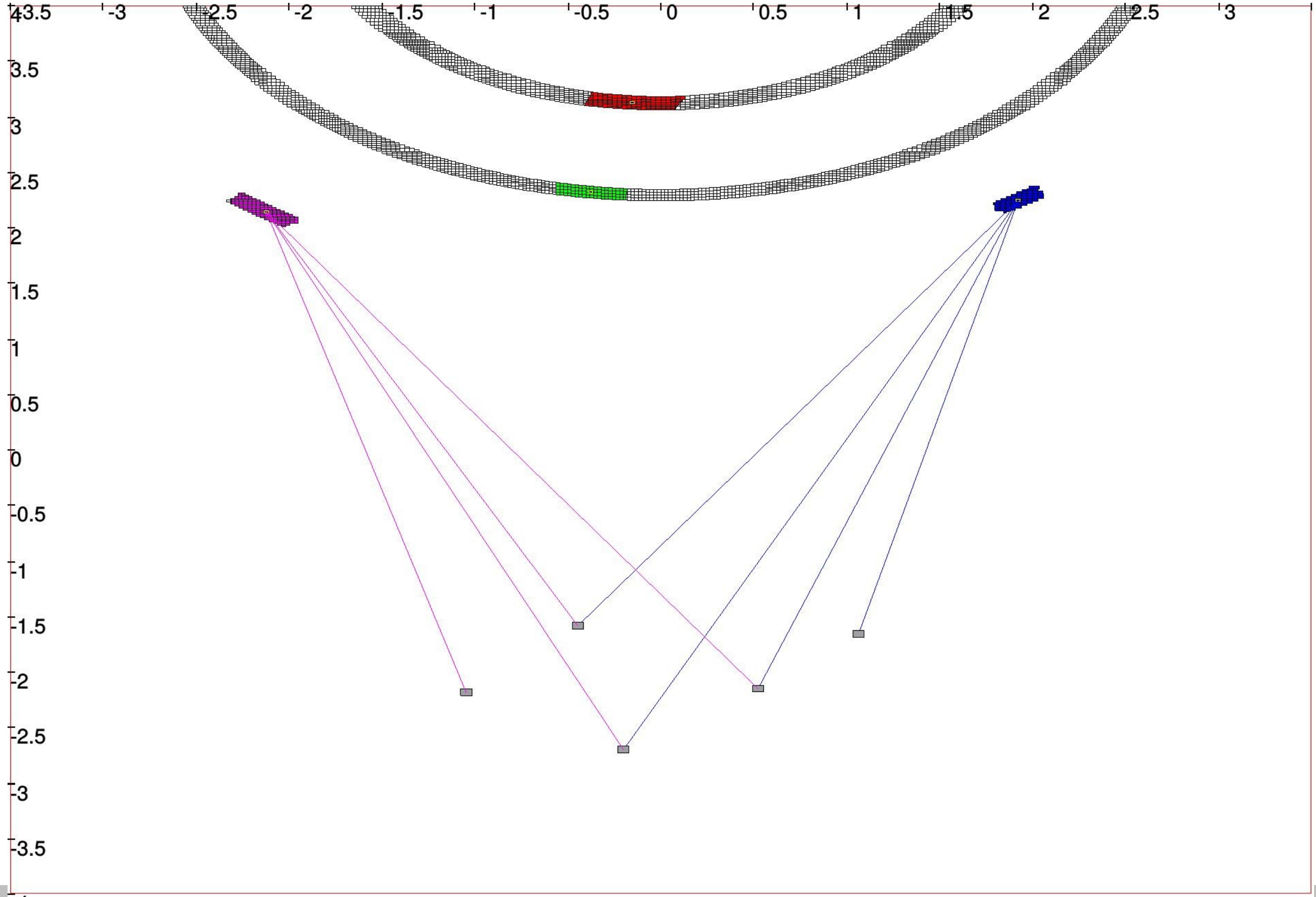
Distributed pose domain characterization

Experimental results : subpaving in full visibility case



Distributed pose domain characterization

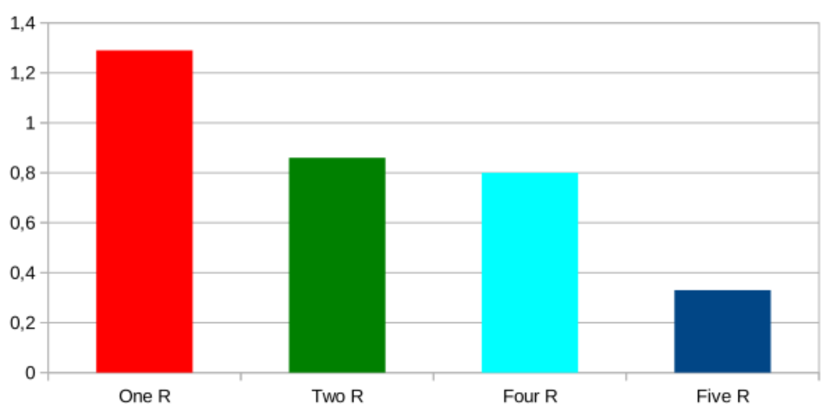
Experimental results : subpaving in reduced visibility case



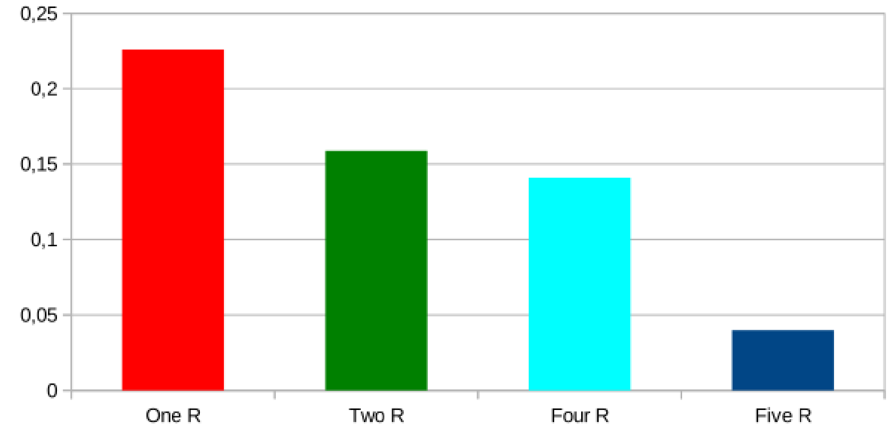
Distributed pose domain characterization

Experimental results : One robot result in the case of 4 robots

Horizontal position domain width (m)



Average horizontal position error width (m)



Conclusion and outlook

- CoB is a good point estimate
- CoB is good initial guess for LM and EKF
- More precise positioning with growing number of robots
- Localization possible in case of complete reduced visibility for some robots due to position exchanges

Thanks for listening !

Question ?