Interval State Estimation With Data Association

B. Desrochers, L. Jaulin

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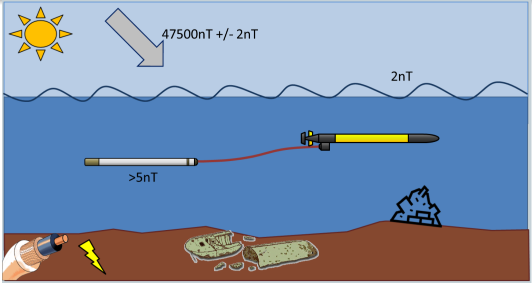
La Cordelière


DGA

Reconstitution de la bataille

youtu.be/yP4cM1UGrqY

Magnetism



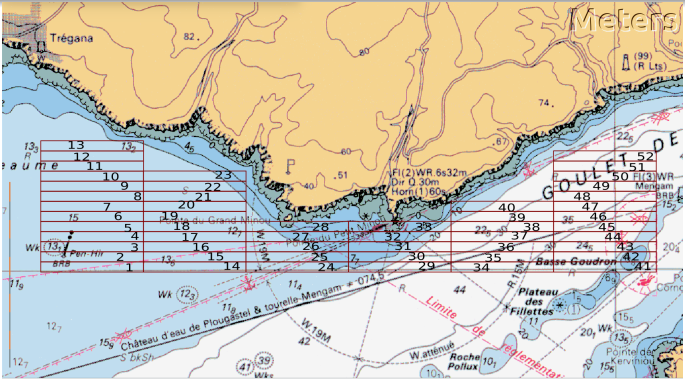
Romain Schwab


aquascan
INTERNATIONAL LTD.

Boatbot


BRE84280
HONDA
caribe
ENSTA

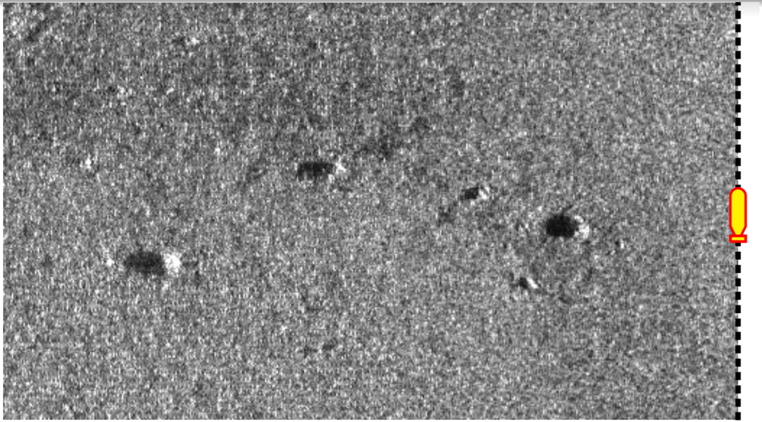

50


Meters
Trégana
GOULET DE
Pointe du Grand Minou
Pointe du Petit Minou
Basse Goudron
Plateau des Fillettes
Château d'eau de Plougastel & tourelle Mengam
Limite de réglementat
Roche Pollux
Pointe de Kerviniou
Fl(2)WR.6s32m
Dir Q.30m
Horn(1)60s
Fl(3)WR
Mengam
Pen-Hir

Robots

Folaga with Kopadia

youtu.be/VqXG9zO_q1A



Formalization

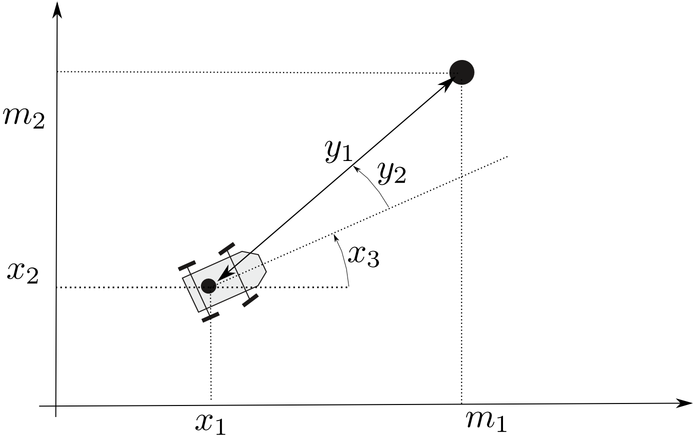
State estimation problem

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)), \mathbf{u}(t)\right) & \text{(evolution equation)} \\ \mathbf{g}(\mathbf{x}(t_i)) \in [\mathbf{y}](t_i) & \text{(observation constraint)} \\ \mathbf{x}(0) \in \mathbb{X}_0 & \text{(initial state)} \end{cases}$$

Implicit form

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)\right), \mathbf{u}(t)) \\ \mathbf{g}(\mathbf{x}(t_i), \mathbf{y}(t_i)) = \mathbf{0}, \\ \mathbf{y}(t_i) \in [\mathbf{y}](t_i) \\ \mathbf{x}(0) \in \mathbb{X}_0 \end{cases}$$

Example. Assume that we have a robot at position (x_1, x_2) with a heading x_3. It measures a landmark located at $(4,5)$.



We have

$$\mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{pmatrix} x_1 + y_1 \cdot \cos(x_3 + y_2) - 4 \\ x_2 + y_1 \cdot \sin(x_3 + y_2) - 5 \end{pmatrix}$$

If now, the robot only measures the distance to the landmark, we get

$$g(\mathbf{x}, y) = (x_1 - 4)^2 + (x_2 - 5)^2 - y^2.$$

In the more general case, the observation constraint is

$$\begin{gathered}\mathbf{g}(\mathbf{x}(t_i), \mathbf{y}(t_i), \mathbf{m}(t_i)) = \mathbf{0} \\ \mathbf{y}(t_i) \in [\mathbf{y}](t_i) \\ \mathbf{m}(t_i) \in [\mathbf{m}](t_i)\end{gathered}$$

In our context, the data are not associated. The observation constraint has the form

$$\mathbf{g}(\mathbf{x}(t_i),\mathbf{y}(t_i),\mathbf{m}(t_i)) = \mathbf{0}$$
$$\mathbf{m}(t_1) \in [\mathbf{m}_1] \vee \cdots \vee \mathbf{m}(t_\ell) \in [\mathbf{m}_\ell]$$

or equivalently

$$\mathbf{g}(\mathbf{x}(t_i),\mathbf{y}(t_i),\mathbf{m}(t_i)) = \mathbf{0}$$
$$\mathbf{m}(t_i) \in \mathbb{M} = [\mathbf{m}_1] \cup \cdots \cup [\mathbf{m}_\ell]$$

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)\right), \mathbf{u}(t)) \\ \mathbf{g}(\mathbf{x}(t_i), \mathbf{y}(t_i), \mathbf{m}(t_i)) = \mathbf{0} \\ \mathbf{y}(t_i) \in [\mathbf{y}](t_i) \\ \mathbf{m}(t_i) \in \mathbb{M} \\ \mathbf{x}(0) \in \mathbb{X}_0 \end{cases}$$

Contractors

The operator $\mathscr{C} : \mathbb{IR}^n \to \mathbb{IR}^n$ is a *contractor* for $f(\mathbf{x}) = 0$, if

$$\begin{cases} \mathscr{C}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)} \\ \mathbf{x} \in [\mathbf{x}] \text{ and } f(\mathbf{x}) = 0 \Rightarrow \mathbf{x} \in \mathscr{C}([\mathbf{x}]) & \text{(consistence)} \end{cases}$$

Building contractors

Consider the equation

$$x_1 + x_2 - x_3 = 0$$

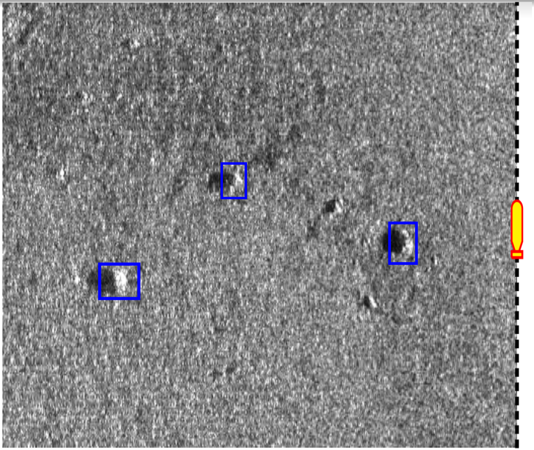
with $x_1 \in [x_1]$, $x_2 \in [x_2]$, $x_3 \in [x_3]$.

We have

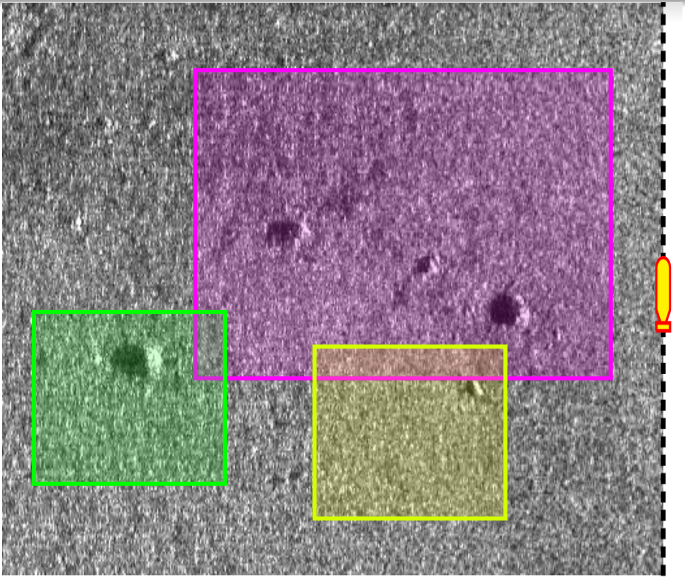
$$\begin{aligned} x_3 = x_1 + x_2 \Rightarrow \quad x_3 \in \quad & [x_3] \cap ([x_1] + [x_2]) \\ x_1 = x_3 - x_2 \Rightarrow \quad x_1 \in \quad & [x_1] \cap ([x_3] - [x_2]) \\ x_2 = x_3 - x_1 \Rightarrow \quad x_2 \in \quad & [x_2] \cap ([x_3] - [x_1]) \end{aligned}$$

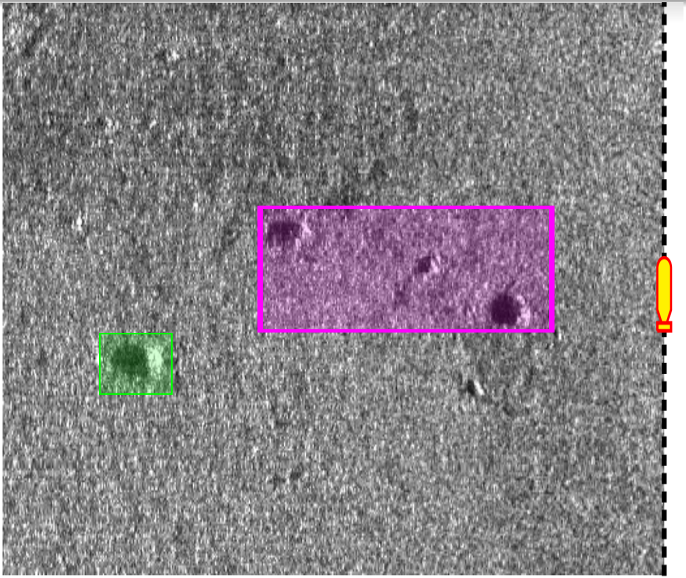
The contractor associated with $x_1 + x_2 = x_3$ is thus

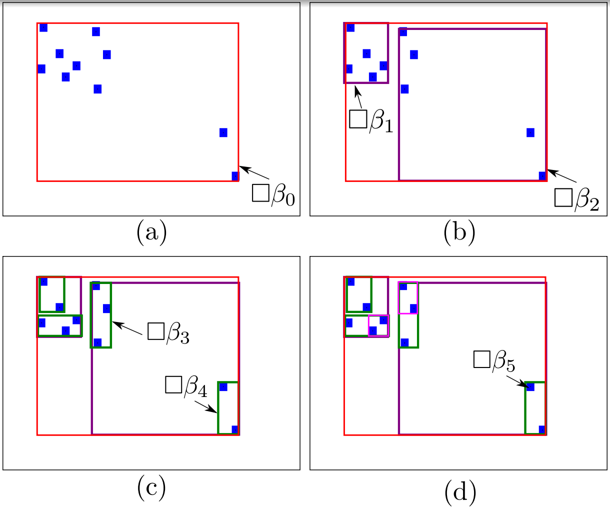
$$\mathscr{C}\begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} = \begin{pmatrix} [x_1]\cap([x_3]-[x_2]) \\ [x_2]\cap([x_3]-[x_1]) \\ [x_3]\cap([x_1]+[x_2]) \end{pmatrix}$$



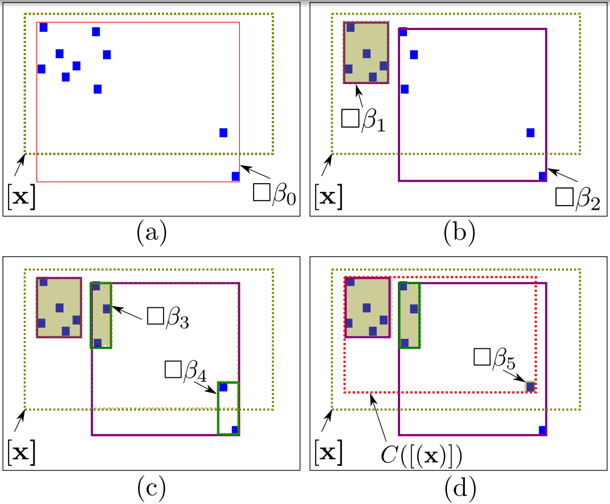
Our constraint :$\mathbf{m} \in \mathbb{M}$





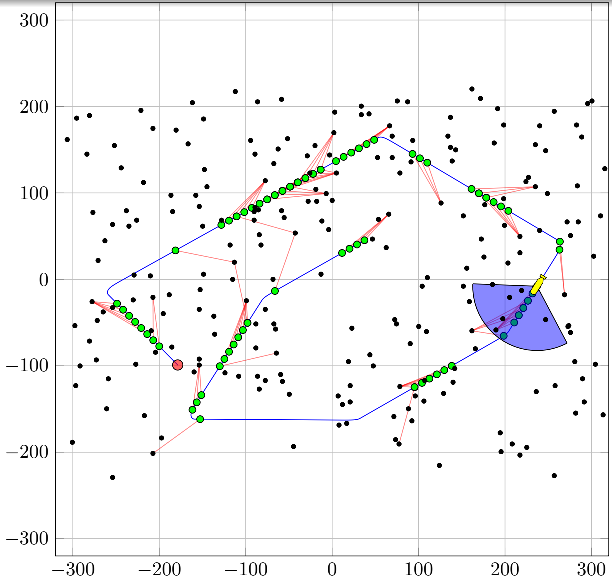


For efficiency, a balanced quadtree is created first



The contractor has now a logarithmic complexity

Test-case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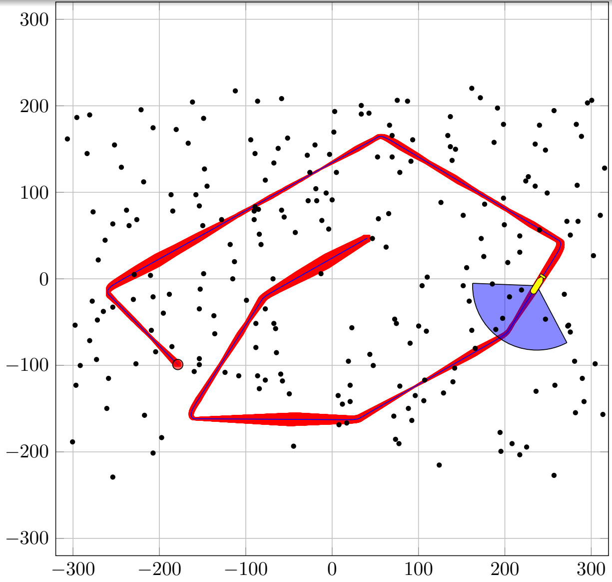
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