Institut für Erdmessung

Leibniz Universität Hannover

Guaranteed Bounding Zones for GNSS Positioning by Geometrical Constraints

11th Summer Workshop on Interval Methods (SWIM 2018)

DFG Research Training Group (GRK2159)
i.c.sens – Integrity and Collaboration in dynamic sensor networks
Leibniz Universität Hannover – Institut für Erdmessung

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Motivation

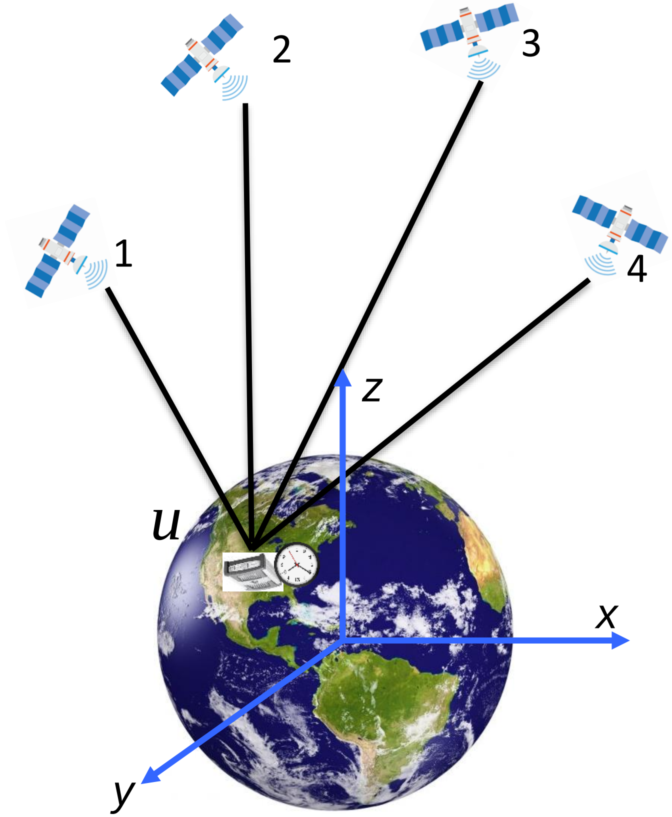
- Geometrical constrains provide rigorous and reliable computing
  - Smallest guaranteed bounding zones
  - Fault detection and exclusion
  - Minimum detectable bias
  - Inconsistency measures

Least Squares Adjustment

- $\rho = \sqrt{(x_{sv} - x_u)^2 + (y_{sv} - y_u)^2 + (z_{sv} - z_u)^2} + \delta t + w = f(x)$

Measurements Knowns Unknowns

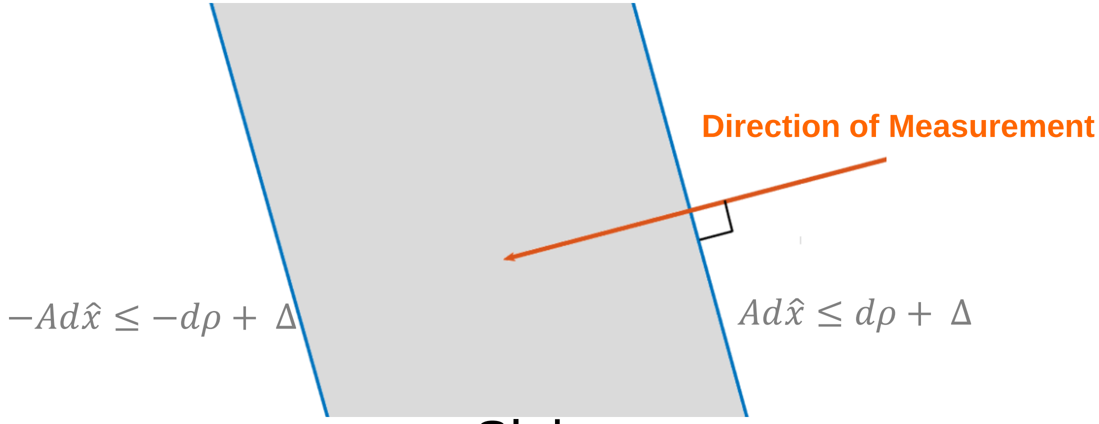
- $$\begin{cases} \rho_1 = \sqrt{(x_{sv1} - x_u)^2 + (y_{sv1} - y_u)^2 + (z_{sv1} - z_u)^2} + \delta t \\ \rho_2 = \sqrt{(x_{sv2} - x_u)^2 + (y_{sv2} - y_u)^2 + (z_{sv2} - z_u)^2} + \delta t \\ \rho_3 = \sqrt{(x_{sv3} - x_u)^2 + (y_{sv3} - y_u)^2 + (z_{sv3} - z_u)^2} + \delta t \\ \rho_4 = \sqrt{(x_{sv4} - x_u)^2 + (y_{sv4} - y_u)^2 + (z_{sv4} - z_u)^2} + \delta t \end{cases}$$



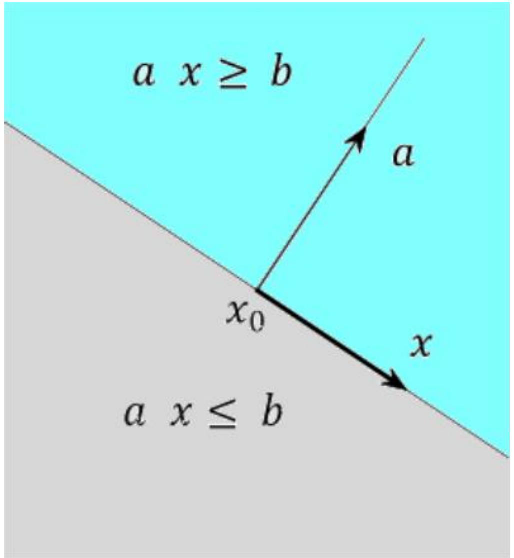
- $\boldsymbol{d\rho} = OMC = f(\boldsymbol{x_0}) - \boldsymbol{\rho} = \boldsymbol{\hat{\rho}} - \boldsymbol{\rho}$
- $\boldsymbol{d\hat{x}} = (\boldsymbol{A^T P A})^{-1} \boldsymbol{A^T P} \cdot \boldsymbol{d\rho}$
- $where\ \boldsymbol{A} = \left[\frac{\partial f}{\partial x}\Big|_{\boldsymbol{x_0}}\right]$
- $\boldsymbol{\hat{x}} = \boldsymbol{x_0} + \boldsymbol{d\hat{x}}$

Primal-Dual Polytope

- Hyperplane is a set of the form $\{x \mid ax = b\}$
- $H - Polytope = \{\boldsymbol{x} \mid \boldsymbol{a_i x} \leq b_i, i = 1, \ldots, n, \boldsymbol{c_i x} = d_i, i = 1, \ldots, p\}$
- $V - Polytope = \text{conv}(\mathbf{X}) = \{\sum_{i=1}^{n} \lambda_i \boldsymbol{x}_i \mid \lambda_i \geq 0, \ \sum_{i=1}^{n} \lambda_i = 1\}$
- $[\boldsymbol{d\rho}] = [\boldsymbol{d\rho} - \Delta, \boldsymbol{d\rho} + \Delta]$



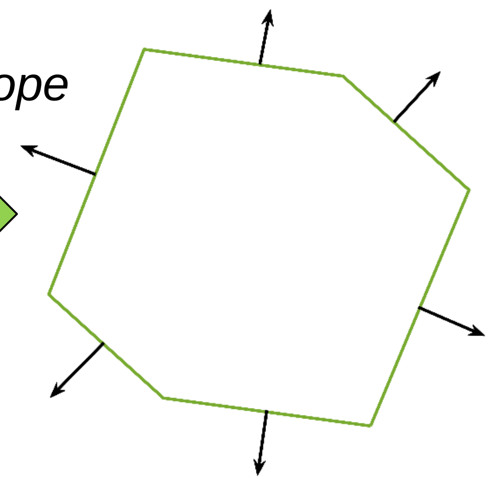
Slab

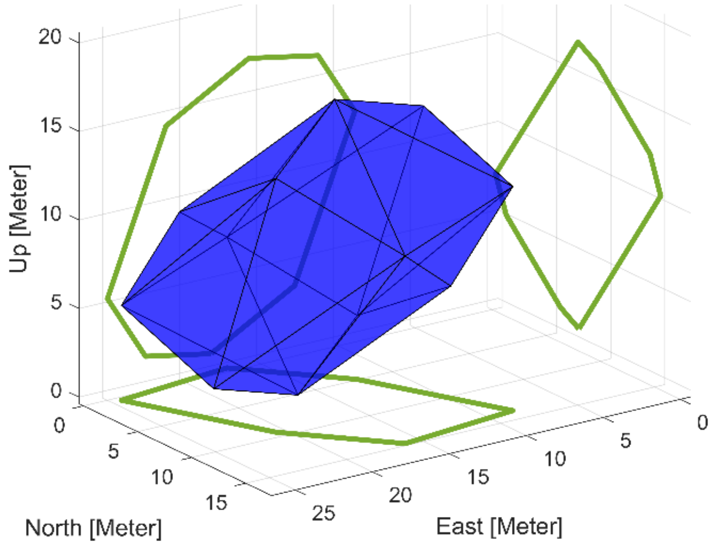


Primal-Dual Polytope

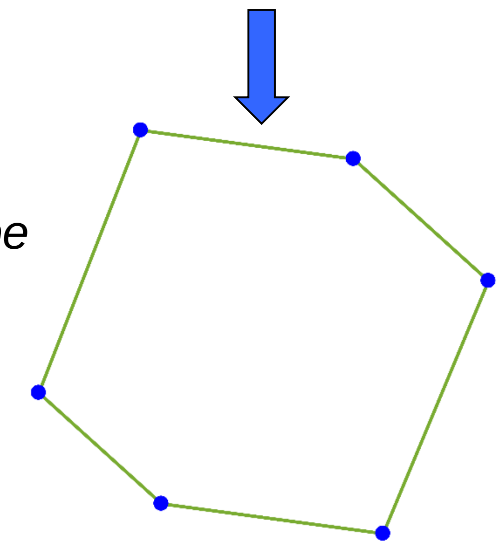
- $\boldsymbol{d\rho} - \Delta \leq \boldsymbol{A d\hat{x}} \leq \boldsymbol{d\rho} + \Delta$
- $\begin{cases} \boldsymbol{A d\hat{x}} \leq \boldsymbol{d\rho} + \Delta \\ -\boldsymbol{A d\hat{x}} \leq -\boldsymbol{d\rho} + \Delta \end{cases} \leftrightarrow \boldsymbol{B d\hat{x}} \leq \boldsymbol{b}$
- $where, \boldsymbol{B} = \begin{bmatrix} \boldsymbol{A} \\ -\boldsymbol{A} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{d\rho} + \Delta \\ -\boldsymbol{d\rho} + \Delta \end{bmatrix}$

H-Polytope





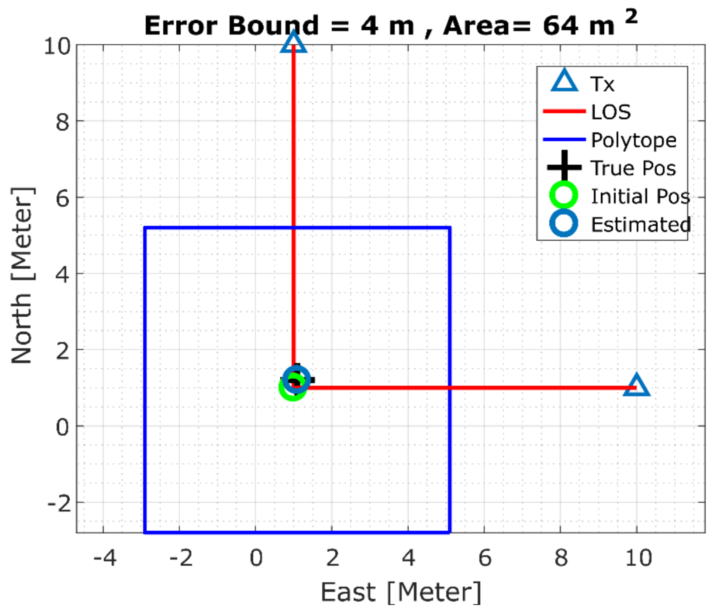
V-Polytope

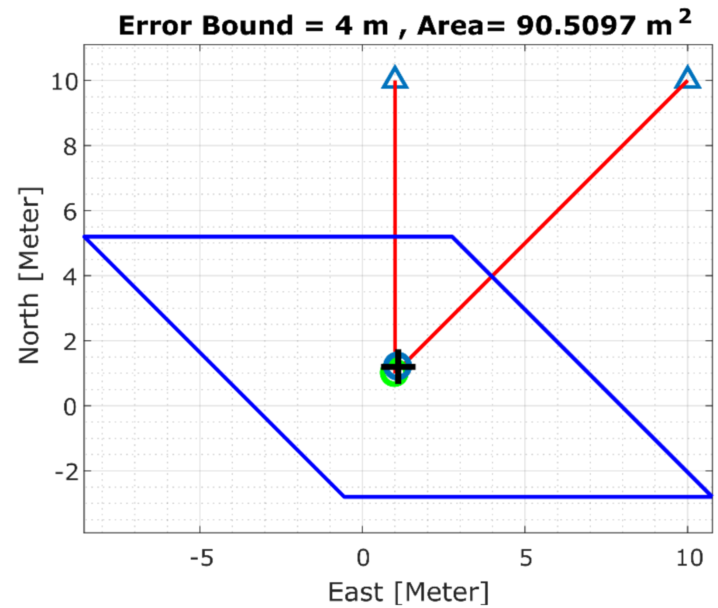


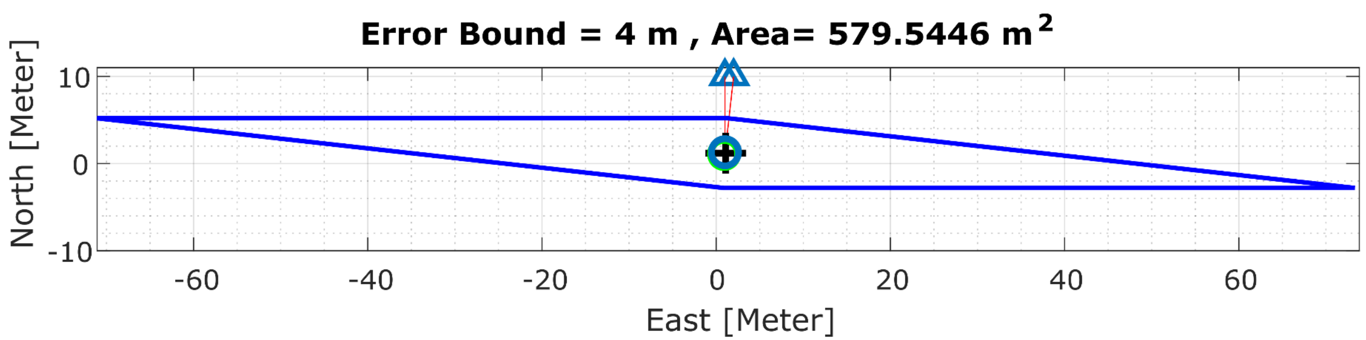
Derivation of Observation Interval Error Bounds

- Three different ways to set the error bounds
  - Probabilistic approach with prior integrity risk
  - Sensitivity analysis of the measurement correction
  - Expert knowledge
- Error bounds and navigation geometry define the volume and the shape of the bounding zone i.e. the polytope

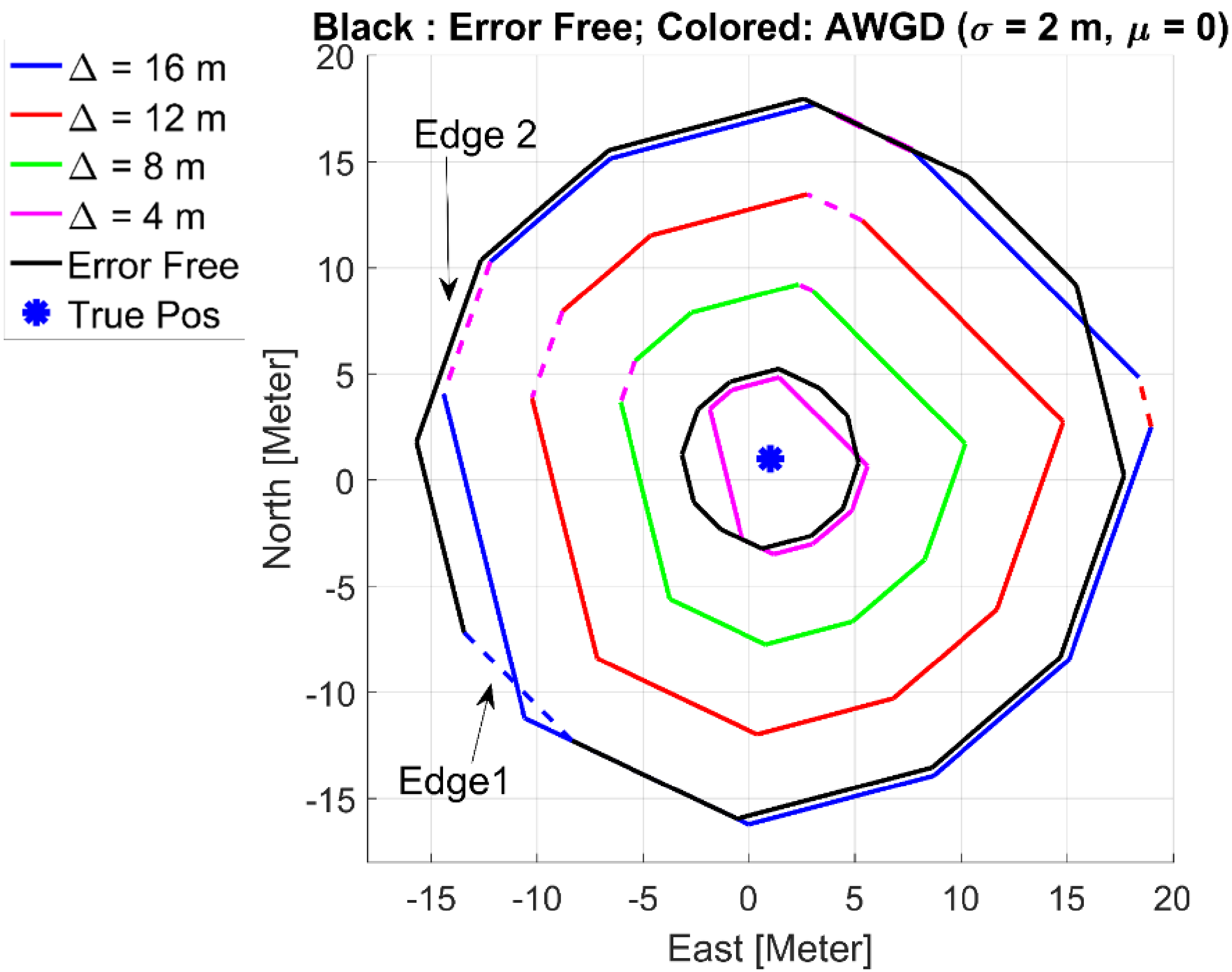
Impact of Geometry





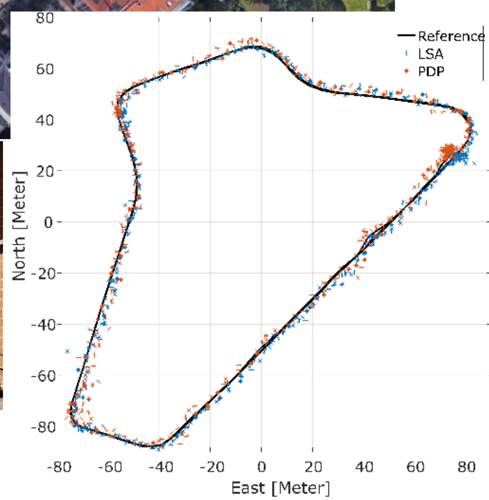


Impact of Random Noise

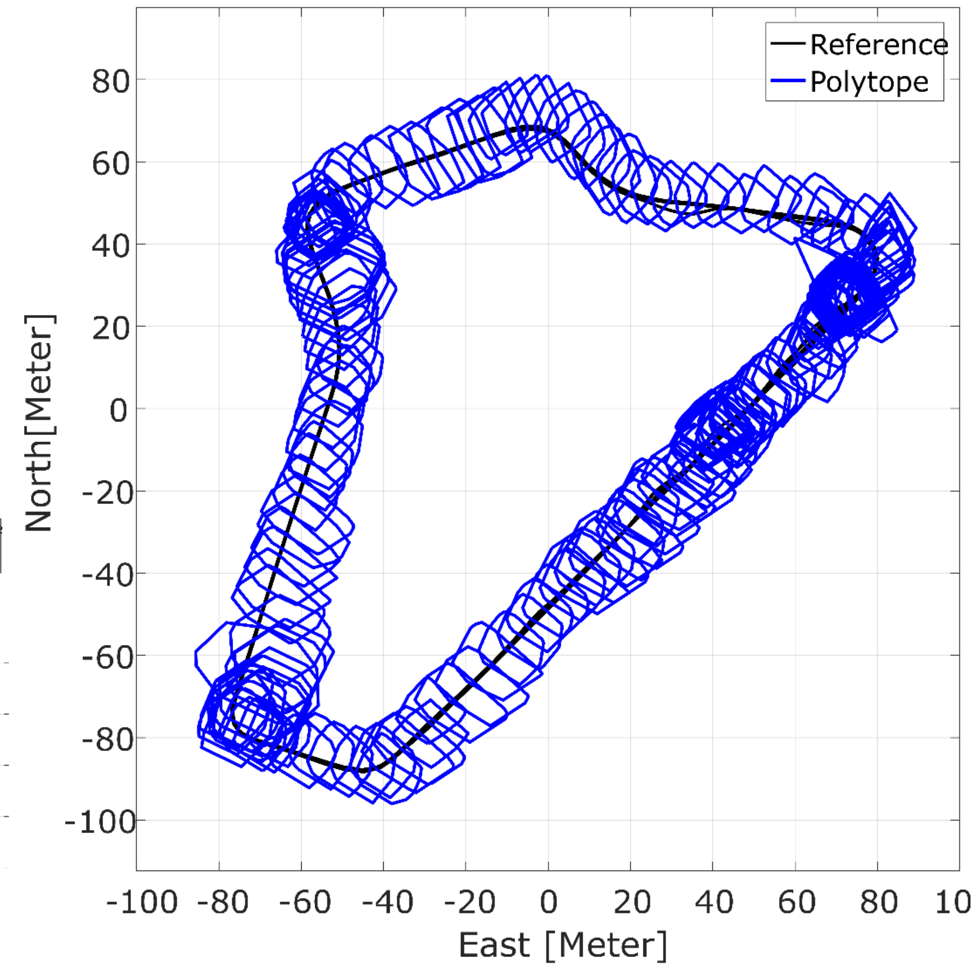


Edges
12
11
10
10
8

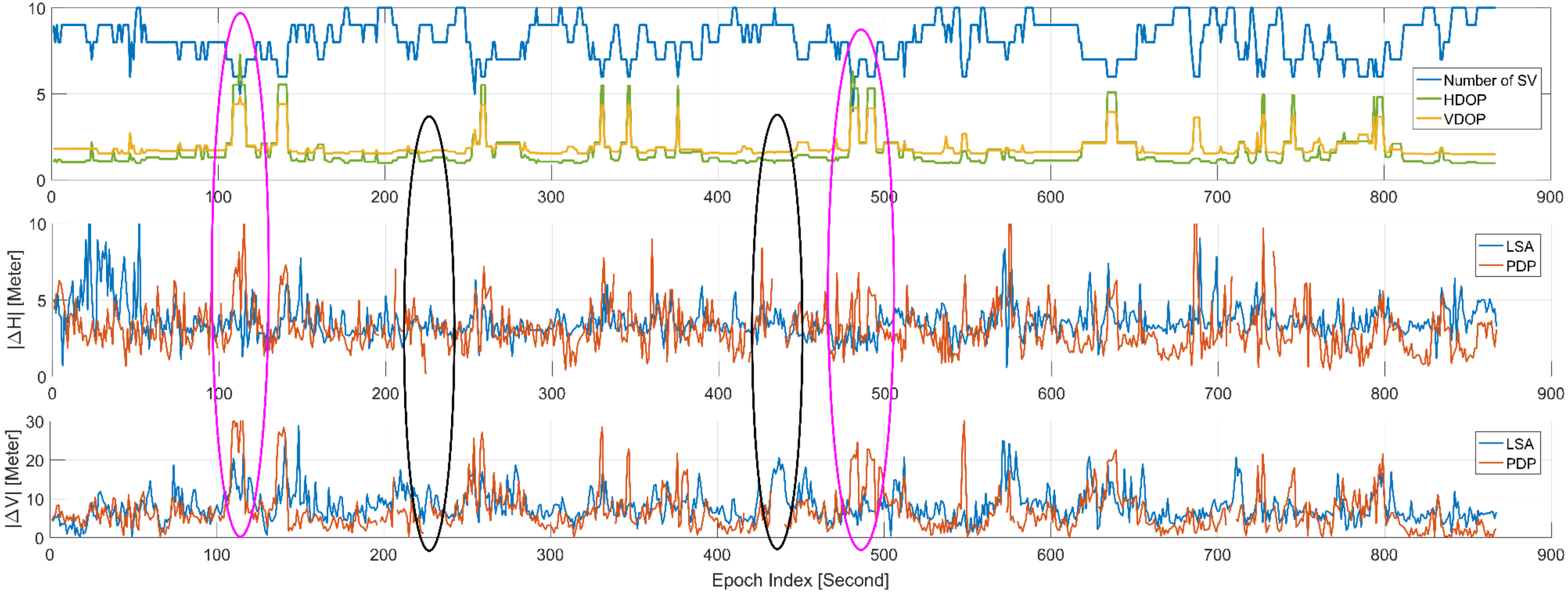
Real Data: Error Indicating Polytope

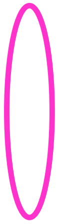


$\Delta = 6m$

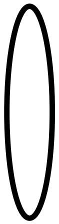


Point Positioning Error Analysis



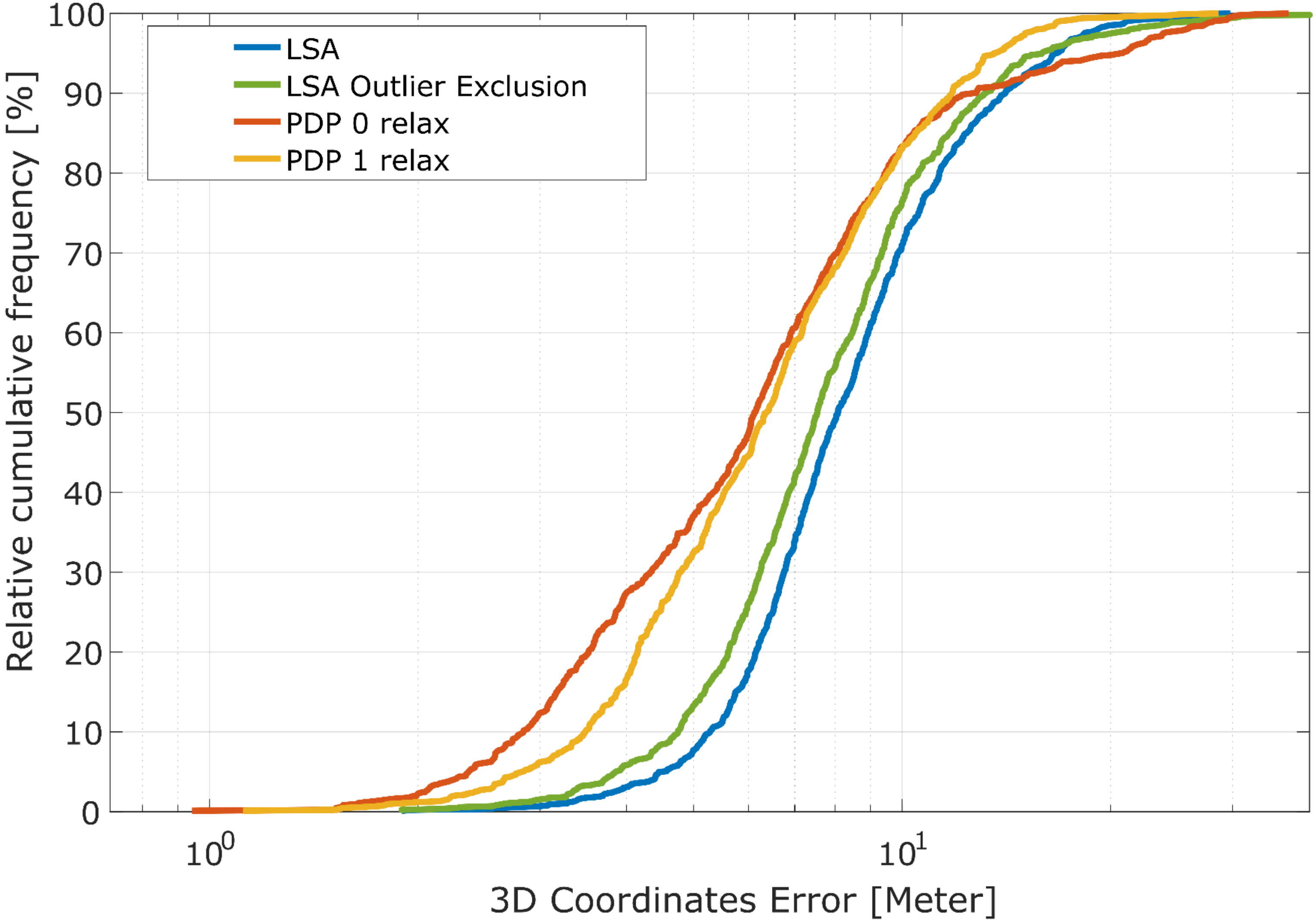


Bad Geometry ➡ high error in PDP

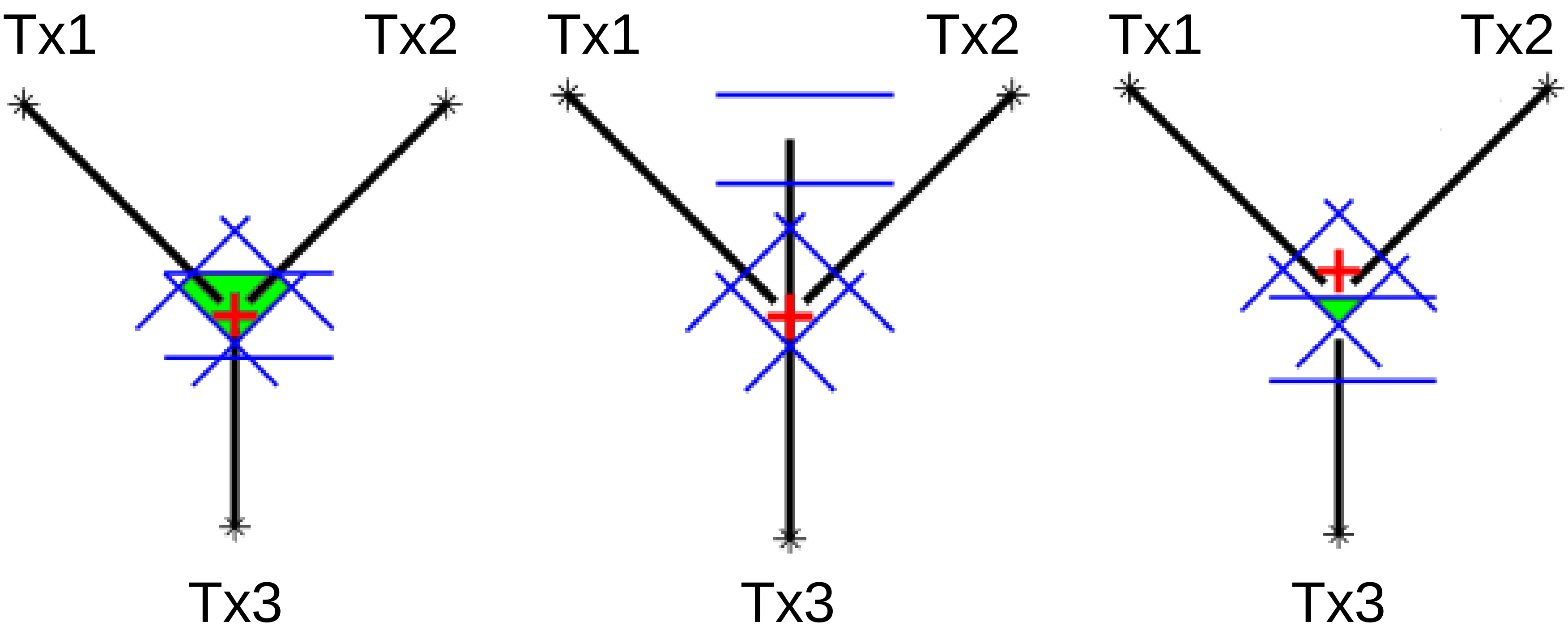


Bias ➡ Empty set from PDP

Point Positioning Error Analysis



Impact of Biased Measurement

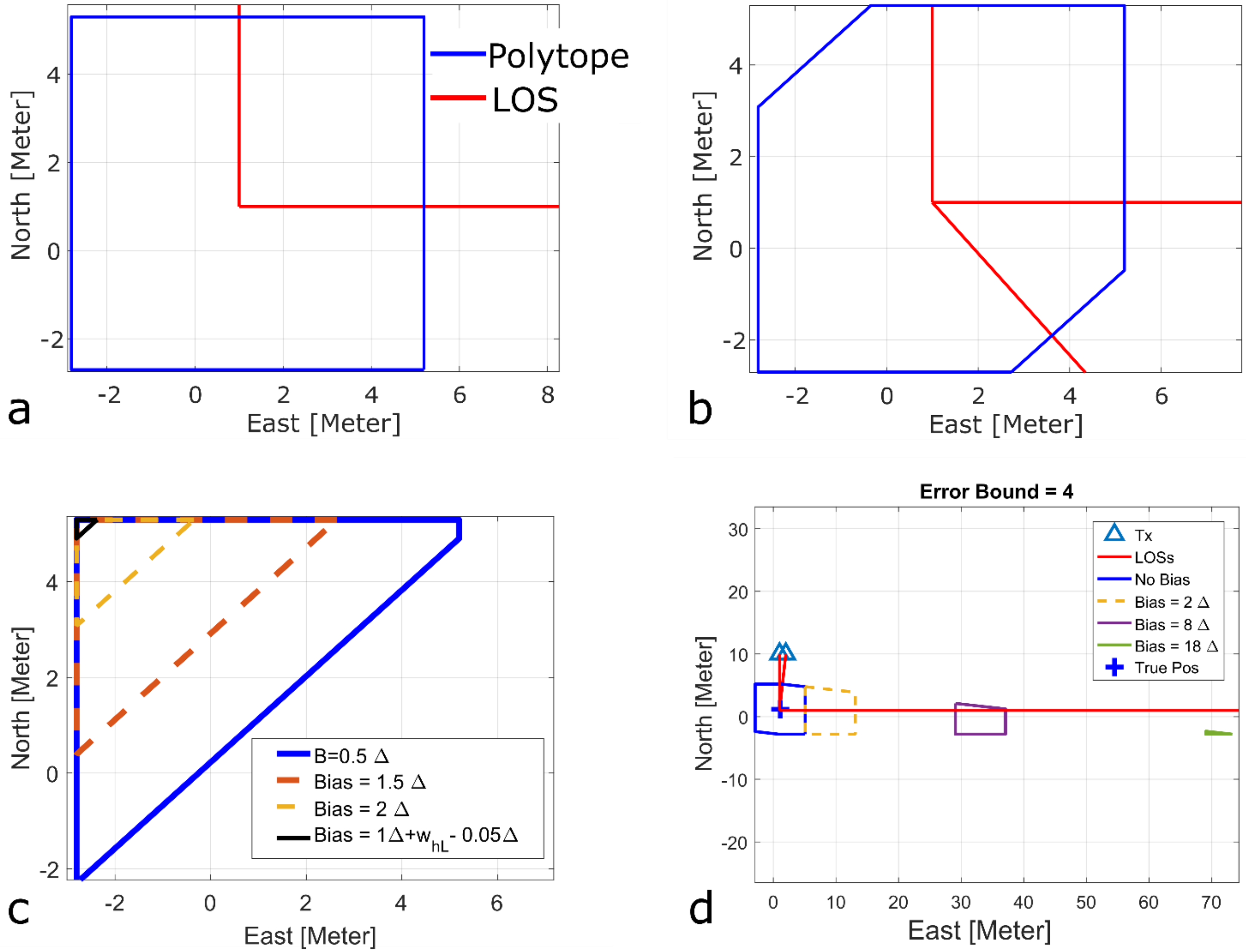


No Bias

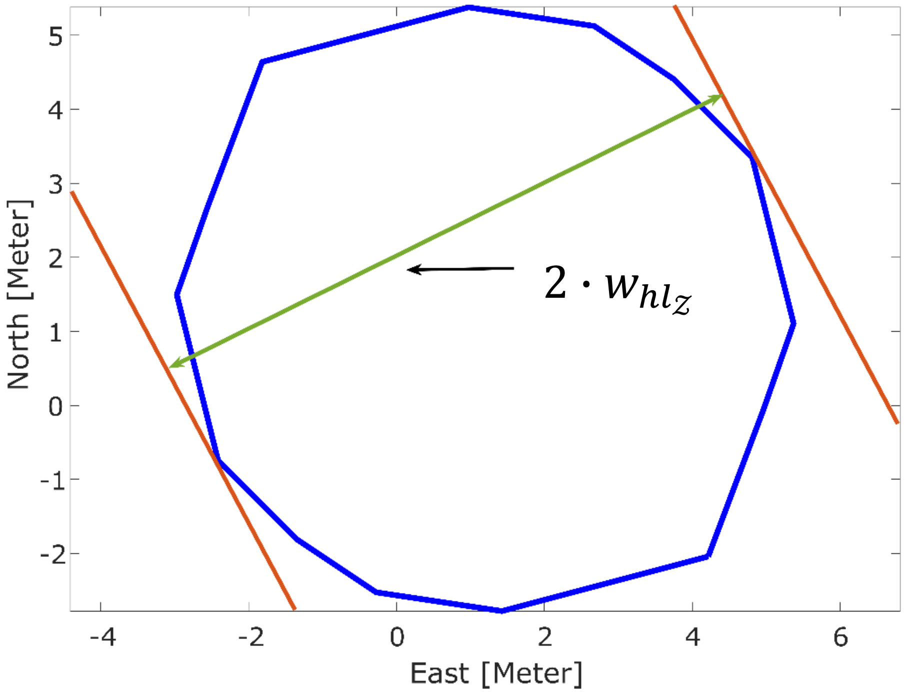
Detectable Bias

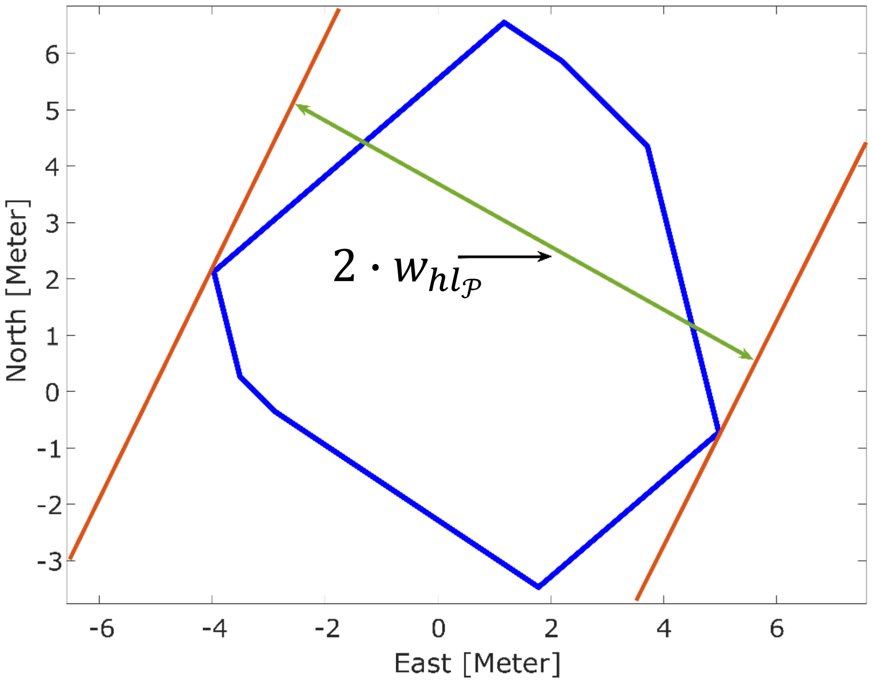
Non-Detectable Bias

Impact of Biased Measurements



Minimum Detectable Bias



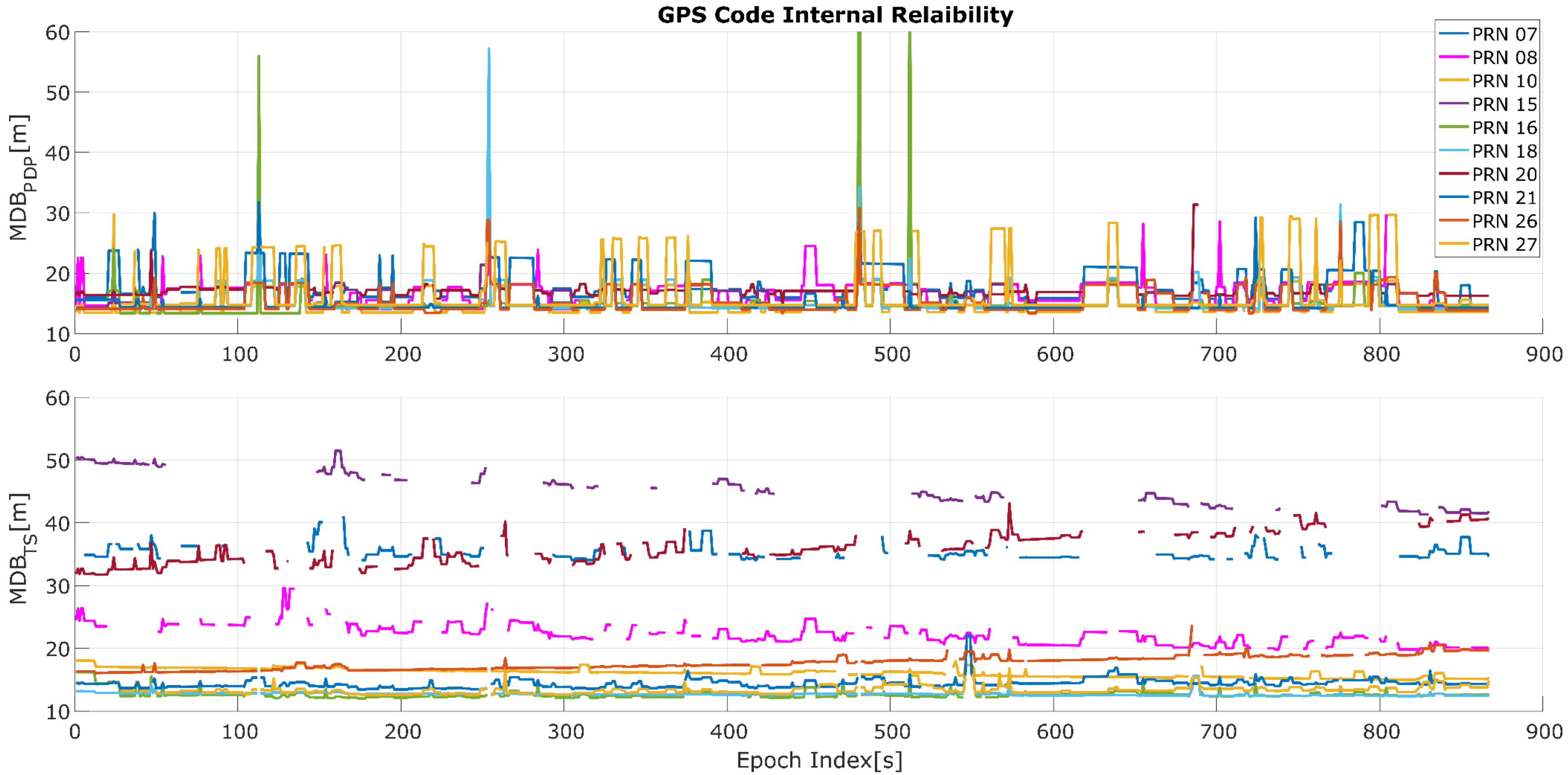


$$MDB_{\mathcal{Z}_i} = w_{hl_{\mathcal{Z}}} + \Delta_i$$

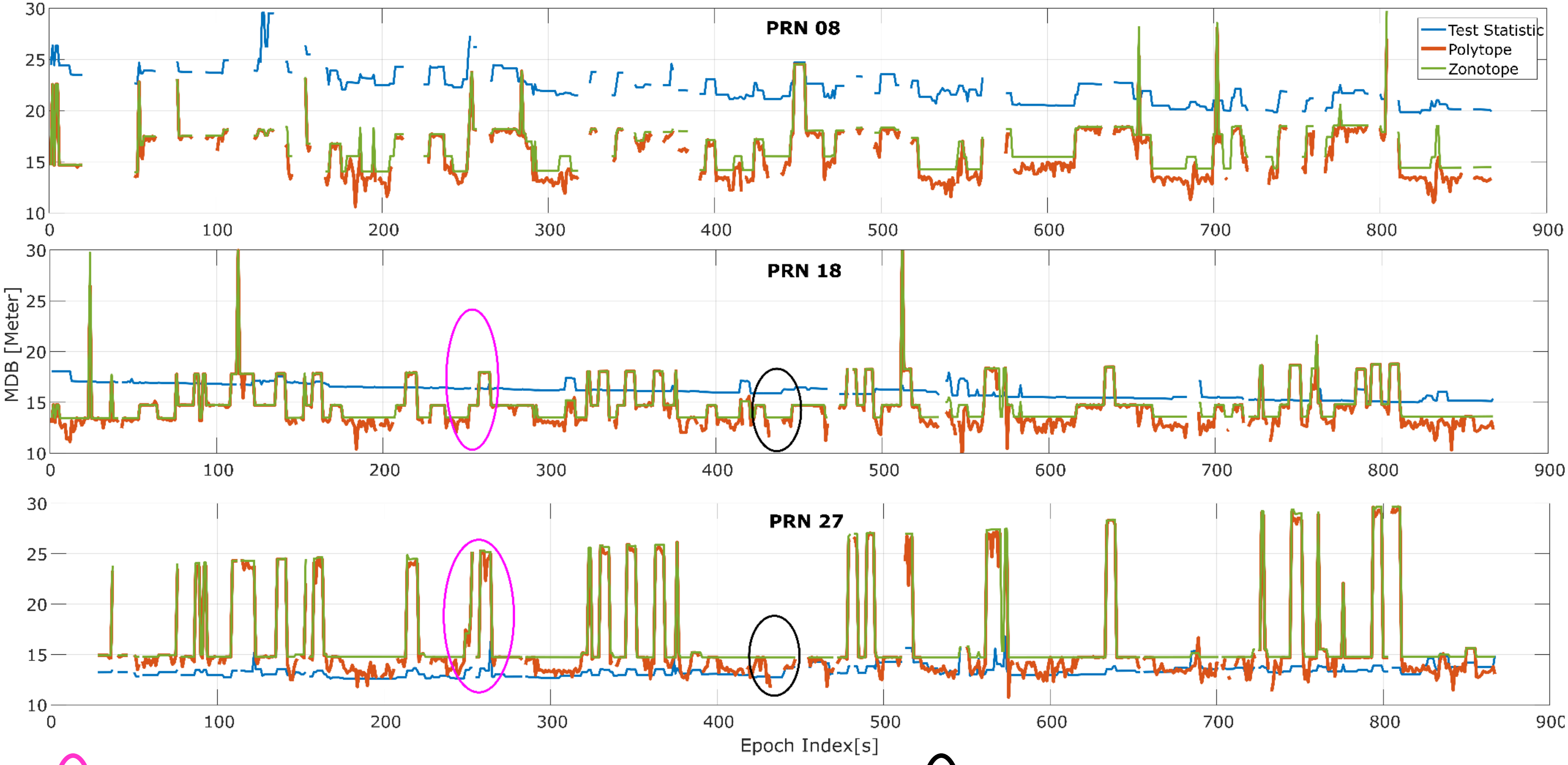
$$MDB_{\mathcal{P}_i} = w_{hl_{\mathcal{P}}} + \Delta_i$$

$$MDB_{TS,i} = \sqrt{\frac{\lambda_0}{c_i^T Q_y^{-1}(I_m - P_A)c_i}}$$

Minimum Detectable Bias



Minimum Detectable Bias

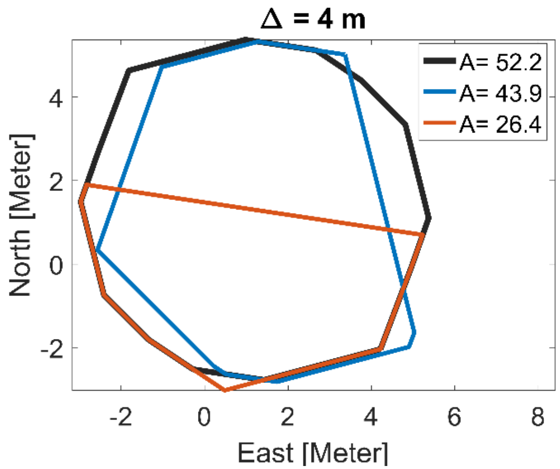


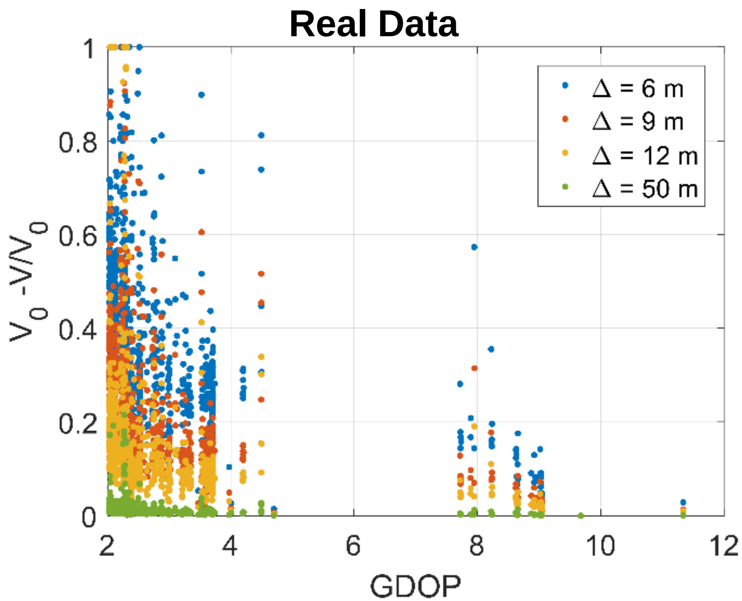
No solution from Test Statistics

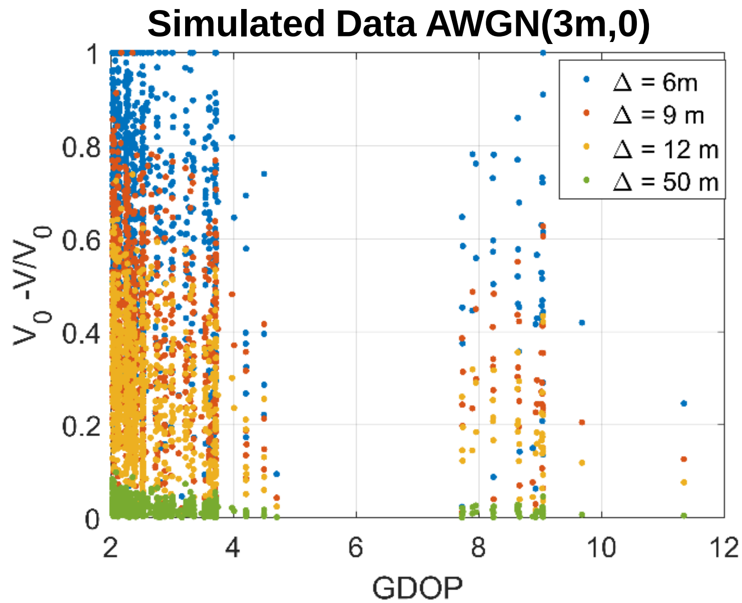
No solution from Polytope

Inconsistency measures

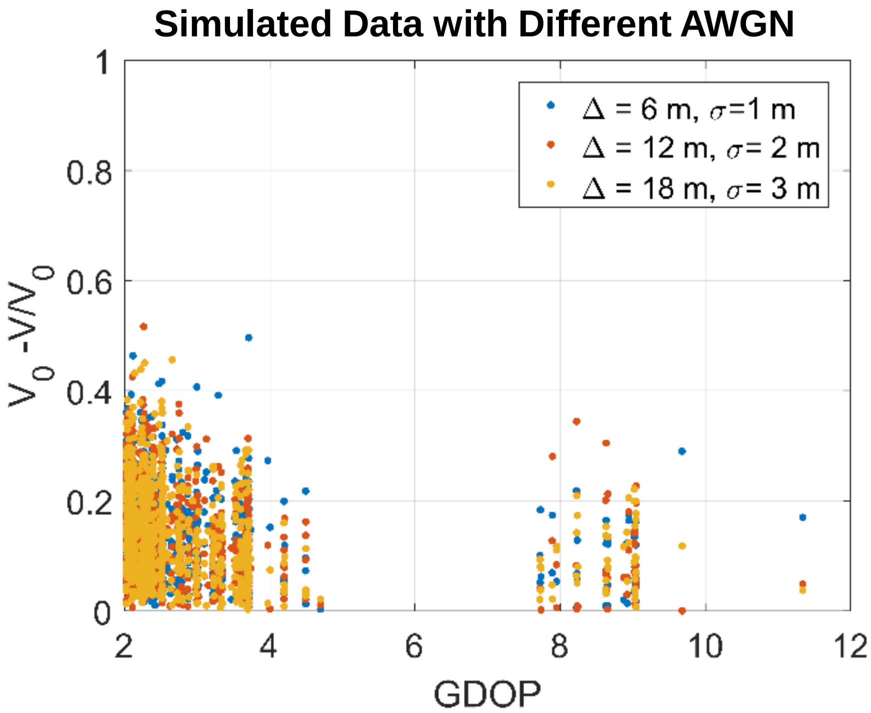
$$Vol_r = \frac{Vol_{\mathcal{Z}} - Vol_{\mathcal{P}}}{Vol_{\mathcal{Z}}} \leq T_r$$

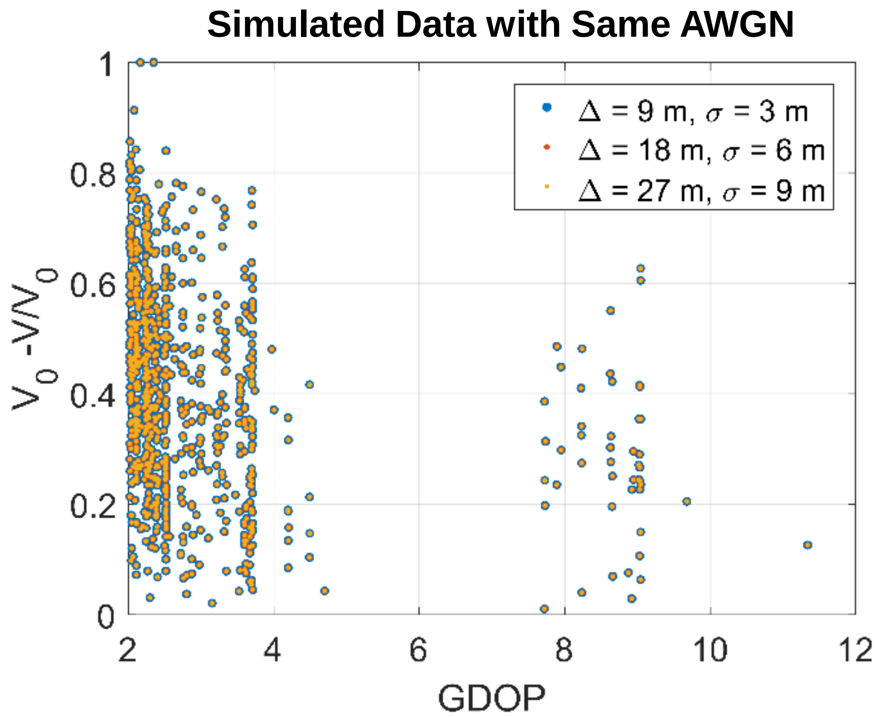




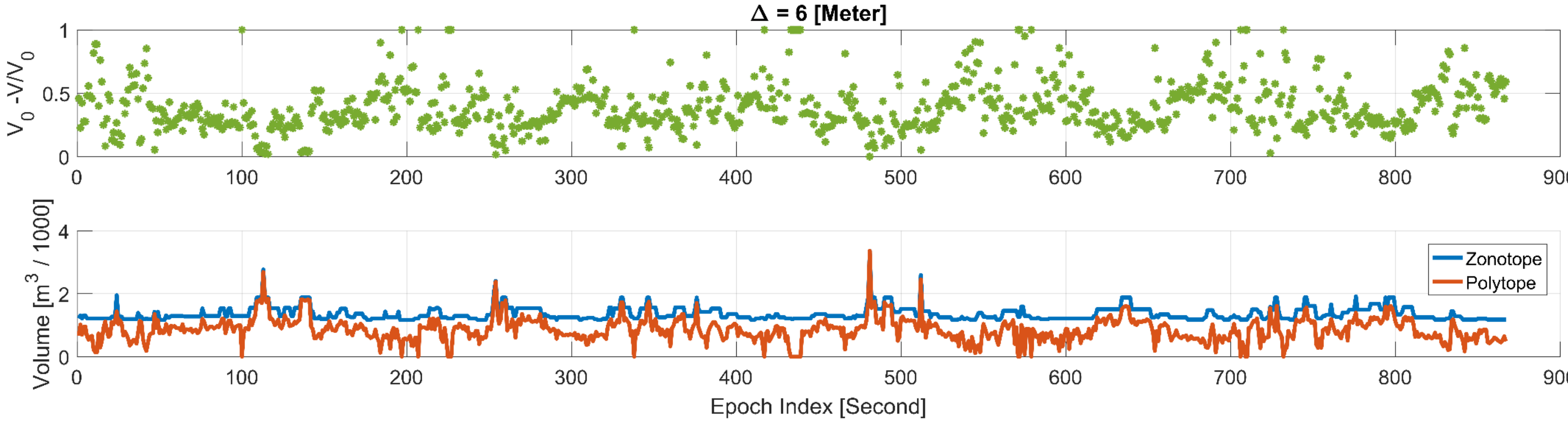


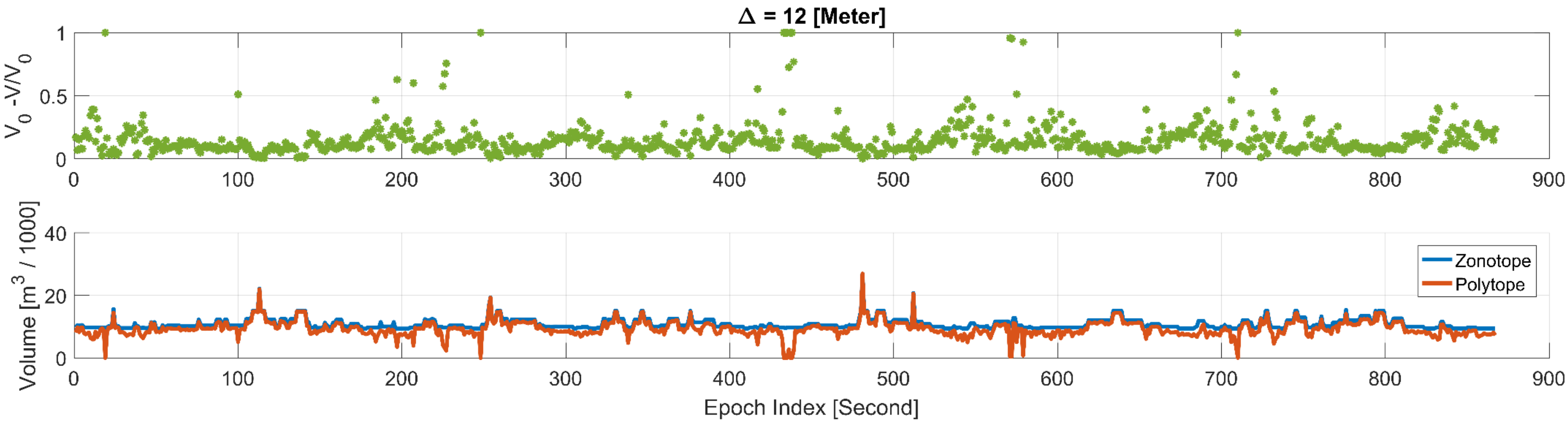
Inconsistency measures





Inconsistency measures





Conclusions

- PDP shows higher precision and accuracy than LSA
- PDP is more sensitive to the positioning geometry than LSA
- New methods to derive MDB with better performance than traditional hypothesis test statistics

- Shape and volume of the polytope are strictly related to the geometry and noise distribution
- PDP gives empty sets in the presence of detectable bias
- PDP provides a inconsistency check bounding zone