A Robust Fault Detection Method using a Zonotopic Kaucher Set-membership Approach - Application to a Real Single-Tank Process

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Institute of Control Systems
Motivation

- State of the art verification
  - Time and resource consuming
  - Based on expert knowledge

- New method needs to handle
  - Increasing system complexity
  - Strict safety requirements
  - Constraints on system dynamics
  - Guarantees

- Idea: Guaranteed verification of system dynamics using inner enclosure
Outline

Main Concept
- Problem setup
- Formalization of the specification
- Measurement assumption
- Guaranteed verification
- Zonotopic enclosure

Examples
- Single Tank Process

Summary
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Main Concept Problem setup

Specification

Formalization

Formal Specification

Measurement

Measurement Data
Main Concept
Formalization of the specification

- System given in ARX form

\[ y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{j=1}^{n_c} c_j u_{k-j} \]

- Input order \( n_a \), output order \( n_c \)
- Output parameter \( a_j \), input parameter \( c_j \)
Main Concept
Formalization of the specification

- An interval specification $S_i^*$ consists of system orders $n_a^*, n_c^* \in \mathbb{R}$ and interval-type model parameters $[a_1^*] \cdots [a_{n_a}^*]$ and $[c_1^*] \cdots [c_{n_c}^*] \in \mathbb{RI}$

- Interval-type parameters allow to express
  - Imprecise or missing knowledge
  - Modelling errors (linear model for nonlinear system)
  - Tolerances

- given by the user
- depicts the combined behavior of plant and controller
Main Concept Problem setup

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Specification

Formalisation

Formal Specification

Implementation

Measurement

Measurement Data

\[ y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{j=1}^{n_c} c_j u_{k-j} \]
Main Concept

Measurement assumption

- Measured input and output data available for verification
- Measurement corrupted by noise

- Interval enclosure of measurement noise available
- Known sensor precision $\delta_{mx}$ as absolute value, with $x \in \{u, y\}$:

$$|\epsilon_{x,k}| \leq |\delta_{mx}|$$

$$[x_k] = [x_{meas,k} - \delta_{mx}, x_{meas,k} + \delta_{mx}]$$
Main concept
Measurement assumption

- Block description

\[
\begin{align*}
[y_{k-1}]a_1 + \cdots + [y_{k-n_a}]a_{n_a} + [u_{k-1}]c_1 + \cdots + [u_{k-n_c}]c_{n_c} &= [y_k] \\
[y_{k+1-1}]a_1 + \cdots + [y_{k+1-n_a}]a_{n_a} + [u_{k+1-1}]c_1 + \cdots + [u_{k+1-n_c}]c_{n_c} &= [y_{k+1}] \\
\vdots & \quad \vdots & \quad \vdots & \quad \vdots & \quad \vdots \\
[y_{T-1}]a_1 + \cdots + [y_{T-n_a}]a_{n_a} + [u_{T-1}]c_1 + \cdots + [u_{T-n_c}]c_{n_c} &= [y_T] \\
\end{align*}
\]

\[ [R]x \\
[d] \]

- Regressor matrix $[R]$, parameter vector $x$, measurement vector $[d]$
Main Concept Problem setup

Specification

Formalisation

Formal Specification

Veriﬁcation

Measurement

Measurement Data

\[ y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{j=1}^{n_c} c_j u_{k-j} \]
Main Concept
Guaranteed verification

- Definition: “Consistency”
  - „The measurement data can be explained by the specification“

Classical interval solution of $[R]x = [d]$ given by interval set $[x]$

- True solution
- Spurious solution
Main concept
Guaranteed verification

- Guaranteed verification
  - No false negatives
  - Only nonspurious behavior
  - Inner enclosure necessary

- United solution set $\sum_{\exists \exists}$
  - $\sum_{\exists \exists}([R], [d]) := \{x \in \mathbb{R} | \exists R \in [R], \exists d \in [d], (Rx = d)\}$
  - Given in Kaucher interval arithmetic

Source: Shary, *Algebraic approach* \(^{[2]}\)

\(^{[2]}\) Shary: Algebraic approach to the interval linear static identification, tolerance, and control problems, or one more application of kaucher arithmetic. *Reliable Computing*, 3-33, 1996.
Main Concept
Verification of direct specification

- Check solution candidate $x \in S_i^*$ using the theorem of Prager-Oettli

$$|R_c x - d_c| \leq R_\Delta |x| + d_\Delta$$

with

- $R_c \in \mathbb{R}^{l \times m}$: $a_c^{(i,j)} = \frac{1}{2} (\bar{a}^{(i,j)} + \underline{a}^{(i,j)})$, center matrix
- $R_\Delta \in \mathbb{R}^{l \times m}$: $a_\Delta^{(i,j)} = \frac{1}{2} (\bar{a}^{(i,j)} - \underline{a}^{(i,j)})$, radius matrix

- Prager-Oettli theorem fulfilled: $x \in S_i^*$ is part of the united solution set $\Sigma_{\exists \exists}$
- Results can be obtained by the evaluation of a single criterion for each solution candidate $x$
Main Concept
Verification of interval specification

- Remaining problem: how to determine the solution candidates to check?

- Check vertexes of specification rectangle
Main Concept
Verification of interval specification

- A parameter vector \( x \in S_i^* \) can explain the measurement if there exists at least one \( \exists R \in [R] \) and at least one \( \exists d \in [d] \) that explains the observation.

- **Full consistency** All parameter vectors \( \forall x \in S_i^* \) can explain the measurement.

- **Basic consistency** At least one parameter vector \( \exists x \in S_i^* \) can explain the measurement.

- **Inconsistency** No parameter vector \( \exists x \in S_i^* \) can explain the measurement.
Main Concept
Verification of interval specification

- Remaining problem: how to determine the solution candidates to check?

- Initial solution: Use vertexes of rectangles\[^1\] in optimization based approach
- Now: Use vertexes of zonotopes\[^2\]


Zonotopic Approach

- Zonotope definition

\[ \Sigma = P_0 \oplus \alpha H_0 K^V = \{ P_0 + \alpha H_0 z : z \in K^V \} \]

- \( \Sigma \) is exhaustively defined by the set of \( v = 1, 2, ..., V \) vertices
- \( P_0 \in \mathbb{R}^{n \times 1} \) is the center of the zonotope
- \( H_0 \in \mathbb{R}^{n \times V} \) is the radius matrix
- \( K^V \) is a unitary box composed of \( V \) unitary interval vectors \( K = [-1,1] \)
Main Concept
Optimization based solution

- Objective function

\[ J(\Sigma_Z) := -\alpha \]

- Find the most possible parameter combinations

- Solution is not unique

- Constraints

- All vertexes \( v = 1,2,\ldots,V \) of the zonotope are part of the united solution

\[ |R_c v - d_c| \leq R_\Delta |v| + d_\Delta \]
Initialize Algorithm

Get new measurement data \( \{u(k), y(k)\} \)

Set up regressor \( \varphi(k) \) and strip \( F_k \)

Calculate OUTER enclosure \( \overline{AFSS}_k \)

Get initial OUTER Zonotope parameter \( p_k \) and \( h_k \)

Start optimization

Minimize \( \alpha \)

Constraint: all vertexes fulfill the PO-theorem

Check: \( \Sigma^O_2 \cap \Theta^* \neq \emptyset \)

A fault might exist

Consistency is proven

No

Yes
Zonotopic Algorithm

Nominal set $\Theta^*$

Feasible set $\Sigma_Z^0 \cap \Theta$

Initial outer Zonotope $\overline{A F S S}_k$

Final inner Zonotope $\Sigma_Z^0$

Strip

Constraints given by the measurement data

- Specified Parameter
Outline

Main Concept
- Problem setup
- Formalization of the specification
- Measurement assumption
- Guaranteed verification
- Zonotopic enclosure

Examples
- Single Tank Process

Summary
Example: Single Tank Process

- Height $h_1$ measured
- Inflow $v_1$ measured

- Tank diameter $154cm^2$
- Pipe diameter $0,5cm^2$
- Leakage diameter $0,8cm^2$
- Max. pump flow $6l/m$
Example: Single Tank Process

- Dynamic of $h_1$:
  \[
  \frac{dh_1}{dt} = -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{\gamma_1 k_1}{A_1} v_1
  \]
  
  outflow \hspace{2cm} \text{inflow by pump 1}

- Time discretization (using $\Delta t = 1$) and transformation to regressor form

  \[
  y(k) = \theta(k) \phi(k):
  \]

  \[
  y(k) = h_1(k) - \frac{\gamma_1 k_1}{A_1} v(k - 1)
  \]

  \[
  \phi(k) = [h_1(k - 1)]
  \]

  \[
  \theta(k) = [A1]
  \]

  with

  \[
  A1(k) = 1 - \frac{a_1}{A_1} \sqrt{\frac{2g}{h_1(k - 1)}}
  \]

  for $h_1 \in [25 \ 45]$ is $A1 \in [0.9712 \ 0.9786]$
Failure free system

\[ \delta_{my} = 0.4 \text{cm} \]
Freeze Failure - Overview

- Failure model

\[ y_{err,k} = y_{k_f} + f_f, \quad \forall k \in [k_f, ..., T] \]

<table>
<thead>
<tr>
<th>Failure ( f_f )</th>
<th>Failure Time ( k_f )</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+2</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+1</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+0.5</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+0.35</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+0.2</td>
<td>60</td>
<td>Not detected</td>
</tr>
</tbody>
</table>
Freeze Failure

$\delta_{my} = 0.4cm$

$ff = 5cm$
Freeze Failure

\[ \delta_{my} = 0.4\text{cm} \]
\[ f_f = 0.35\text{cm} \]
Offset Failure - Overview

- Failure model

\[ y_{err,k} = y_k + f_o, \quad \forall k \in [k_f, \ldots, T] \]

<table>
<thead>
<tr>
<th>Failure ( f_o )</th>
<th>Failure Time ( k_f )</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+2</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+1</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+0.5</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+0.35</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>+0.2</td>
<td>60</td>
<td>Not detected</td>
</tr>
</tbody>
</table>
Offset failure

\[ \delta_{my} = 0.4\text{cm} \]

\[ f_o = 5\text{cm} \]
\[ \delta_{my} = 0.4 \text{cm} \]
\[ f_o = 0.35 \text{cm} \]
Scaling Failure - Overview

- Failure model

\[ y_{\text{err},k} = y_k \cdot f_s, \quad \forall k \in [k_f, \ldots, T] \]

<table>
<thead>
<tr>
<th>Failure $f_s$</th>
<th>Failure Time $k_f$</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>0.75</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>0.9</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>0.95</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>0.97</td>
<td>60</td>
<td>Not detected</td>
</tr>
<tr>
<td>1.01</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>1.03</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>1.05</td>
<td>60</td>
<td>Detected</td>
</tr>
<tr>
<td>1.1</td>
<td>60</td>
<td>Detected</td>
</tr>
</tbody>
</table>
Offset failure

\[
\delta_{my} = 0.4 \text{cm} \\
\hat{f}_s = 0.5
\]
Offset failure

\[ \delta_{my} = 0.4 \text{cm} \]
\[ f_s = 0.95 \]
Offset failure

\[ \delta_{my} = 0.4 \text{ cm} \]

\[ f_s = 1.01 \]
Summary

- Formalized specification defined
  - Interval specification $S_i^*$
  - Noise modelling
- Guaranteed verification based on united solution set
- Optimization based verification
  - Pointwise verification of vertexes
  - Zonotopic enclosure
- Application to a single tank process
  - Very small failures can be detected
Thank you for your kind attention